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International Journal of Electrical and Electronics Research (IJEER)
Research Article | Volume 10, Issue 4 | Pages 837-842 | e-ISSN: 2347-470X

Fractional-order Diffusion based Image Denoising Model

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ABSTRACT- Edge indicating operators such as gradient, mean curvature, and Gauss curvature-based image noise removal algorithms are incapable of classifying edges, ramps, and flat areas adequately. These operators are often affected by the loss of fine textures. In this paper, these problems are addressed and proposed a new coefficient of diffusion for noise removal. This new coefficient consists of two edge indicating operators, namely fractional-order difference curvature and fractional-order gradient. The fractional-order difference curvature is capable of analyzing flat surfaces, edges, ramps, and tiny textures. The fractional-order gradient can able to distinguish texture regions. The selection of the order is more flexible for the fractional order gradient and fractional-order difference curvature. This will result in effective image denoising. Since the discrete Fourier transform is simple to numerically implement, it is taken into consideration for the implementation of fractional-order gradient. The proposed method can give results that are visually appealing and improved quantitative outputs in terms of the Figure of Merit (FoM), Mean Structural Similarity (MSSIM), and Peak Signal to Noise Ratio (PSNR), according to comparative analysis.

Keywords: Anisotropic Diffusion, Difference Curvature, Fourier Transform, Fractional Derivative, Image Denoising.

ARTICLE INFORMATION

Author(s): Sridevi Gamini, Vishnu Vardhan Gudla and Ch Hima Bindu;

Received: 08/08/2022; **Accepted**: 06/10/2022; **Published**: 18/10/2022;

e-ISSN: 2347-470X;

Paper Id: IJEER-RDEC5973;

Citation: 10.37391/IJEER.100413

Webpage-link:

https://ijeer.forexjournal.co.in/archive/volume-10/ijeer-100413.html

crossref member CROSSREF.ORG THE CITATION LINKING BACKBONE

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1. INTRODUCTION

The development of an image denoising model that eliminates noise while maintaining crucial features, such as texture details and edges, is a challenging task. Image denoising can be modelled mathematically as an ill-posed inverse problem. A classical method for the inverse problem is variational method and it is based on energy minimization. This method can be used to model the image in the bounded variation (BV) function space while keeping the image edges preserved. The functions in BV space are allowed to have jumps and interruptions of discontinuous information, even if the edges are well represented. The functions are piece-wise continuous and the solutions are frequently piece-wise constant, resulting in staircase effects in image reconstruction. This approach has been generally observed in various image processing approaches with diverse applications, such as image super-resolution, image restoration, image inpainting, and image denoising.

A variety of noise removal methods using integer-order partial differential equations (PDE) [8, 13, 14] are available in the literature. Among these energy minimization approaches, anisotropic diffusion introduced by Perona and Malik (PM) [14] and variational approach introduced by Rudin et al. [13] are very popular for the edge preservation capability. It is noted that the PM method produces encouraging solutions for the elimination of noise in the images while keeping edges of an object and minimizing oscillations. However, it retains some unacceptable effects, such as, lose in contrast and texture details also produces staircase effects [3,9]. You and Kaveh (YK) proposed a 4th- order PDE-based approach for the removal of noise [12]. The absolute value of Laplacian is defined for diffusion coefficient. This approach can overcome the staircase effect which is introduced by the PM approach. However, it has the weak holding ability of preserving the edges and tends to leave the processed image with speckle noise.

When the image is with variety of structures, such as, small-scale textures, ramps, edges, corners, lines, flat regions, and so on, the gradient features are not sufficient to represent the complex structure of an image. So, many researchers worked on the new geometric features (curvature, structure tensor, etc.) for the image restoration.

Chan and Shen introduced Mean curvature-driven diffusion (MCDD) model [11] for the image reconstruction. This MCDD model is an advancement of anisotropic diffusion, where mean curvature acts as a conductance coefficient. As a result, the illumination is flattened and smoothed out. In 2005 [9], the authors proposed a diffusion model driven by Gauss curvature. The Gauss curvature operator cannot differentiate edges from ramps and flat areas. Further, isolated noise and edges cannot be distinguished by the mean curvature operator. While the previous models can preserve the edges and ramps, they may introduce edge blurriness and texture fuzziness.

The fractional-order calculus has been widely used in image processing to retain the edges [1-7]. The "non-local" attribute of the fractional-order differentiation operator allows for improved texture preservation, in contrast to the traditional integer-order differentiation operator. This paper proposes the fractional-order gradient-based diffusion coefficient and the fractional-order difference curvature (fDC). This new conduction coefficient can well differentiate edges of an image from the flat and ramp areas. The suggested model incorporates fractional-order gradients into anisotropic diffusion, which is a natural interpolation between the second-order (PM) and fourth-order diffusion models. The improved conduction coefficient successfully retains the edges of an image and lessens the staircase effect by utilizing the various diffusion speed benefits in various parts of the conduction function.

The article is presented as follows. *Section 2* explains fractional-order derivative using DFT. A new image denoising model is presented by using fractional anisotropic diffusion and fractional-order gradient and is explained in *Section 3*. In *Section 4*, the performance of the suggested model and state-of-art image denoising models is compared and examined. *Section 5* provides conclusions.

2. FRACTIONAL ORDER DERIVATIVE USING DFT

The generalized form of the integer-order derivative is referred as the fractional-order derivative. There are different definitions given by many mathematicians to fractional-order derivative which give different results. In the applications of image processing, the definition of fractional-order derivative using discrete Fourier transform (DFT) is considered since it is simple to execute [3]. The 2-D DFT of an image $u(x, y)$ of $M \times M$ size is represented as

$$\hat{u}(\omega_1, \omega_2) = \frac{1}{M^2} \sum_{x,y=0}^{M-1} u(x, y) e^{-\frac{j2\pi(\omega_1 x + \omega_2 y)}{M}} \quad (1)$$

For 2-D DFT, the translation property in spatial-domain can be denoted as

$$u(x - x_0, y - y_0) \overset{F}{\leftrightarrow} e^{-\frac{j2\pi(\omega_1 x_0 + \omega_2 y_0)}{M}} \hat{u}(\omega_1, \omega_2) \quad (2)$$

Where, F is 2D-DFT. The first-order partial difference in the x direction can therefore be written as

$$D_x u(x, y) = u(x, y) - u(x - 1, y) \quad (3)$$

$$D_x u(x, y) \overset{F}{\leftrightarrow} \left(1 - e^{-\frac{j2\pi\omega_1}{M}}\right) \hat{u}(\omega_1, \omega_2) \quad (4)$$

and the DFT of fractional-order partial difference in the x-direction is denoted by

$$D_x^\alpha u(x, y) \overset{F}{\leftrightarrow} \left(1 - e^{-\frac{j2\pi\omega_1}{M}}\right)^\alpha \hat{u}(\omega_1, \omega_2) \quad (5)$$

Similarly, the DFT of fractional-order partial difference in the y-direction denoted by

$$D_y^\alpha u(x, y) \overset{F}{\leftrightarrow} \left(1 - e^{-\frac{j2\pi\omega_2}{M}}\right)^\alpha \hat{u}(\omega_1, \omega_2) \quad (6)$$

In general, the fractional-order derivative operator of two-dimensional signal can be written as

$$D^\alpha u(x, y) = \left(D_x^\alpha u(x, y), D_y^\alpha u(x, y)\right) \quad (7)$$

And

$$|D^\alpha u(x, y)| = \sqrt{(D_x^\alpha u(x, y))^2 + (D_y^\alpha u(x, y))^2} \quad (8)$$

The central difference approach is highly helpful in actual computations to calculate the fractional-order difference. This is analogous to shifting of $D_x^\alpha u(x, y)$ and $D_y^\alpha u(x, y)$ by $\alpha/2$ units.

$$\widetilde{D}_x^\alpha u(x,y) = F^{-1}\left(\left(1 - e^{-\frac{j2\pi\omega_1}{M}}\right)^\alpha e^{\frac{j\pi\alpha\omega_1}{M}} \hat{u}(\omega_1, \omega_2)\right) \quad (9)$$

$$\widetilde{D}_y^\alpha u(x, y) = F^{-1}\left(\left(1 - e^{-\frac{j2\pi\omega_2}{M}}\right)^\alpha e^{\frac{j\pi\alpha\omega_2}{M}} \hat{u}(\omega_1, \omega_2)\right) \quad (10)$$

where, F^{-1} is the 2-D inverse discrete Fourier transform (IDFT). The operator $\widetilde{D}_x^\alpha$ has the form $[F^{-1}][K_1][F]$, where [.] is a matrix operator, and

$$K_1 = diag\left(\left(1 - e^{-\frac{j2\pi\omega_1}{M}}\right)^\alpha e^{\frac{j\pi\alpha\omega_1}{M}}\right) \quad (11)$$

The following ocncept can be used to calculate the adjoint operator $\widetilde{D}_x^{\alpha^*}$ of $\widetilde{D}_x^\alpha$

$$\widetilde{D}_x^{\alpha^*} = ([F^{-1}][K_1][F])^* = [F][K_1^*][F^{-1}] \quad (12)$$

Since K_1 is purely diagonal operator, K_1^* is the complex conjugation of K_1. The same procedure can be used for the calculations of $\widetilde{D}_y^{\alpha^*}$ and $\widetilde{D}_y^\alpha$.

$$\widetilde{D}_y^{\alpha^*} = [F][K_2^*][F^{-1}] \quad (13)$$

3. PROPOSED MODEL

In this article, a new fractional - order PDE for the removal of noise is presented. Initially, for the effective image denoising, the advantage of fractional-order gradient and fractional-order difference curvature is considered. Fractional-order gradient $(D^\alpha u)$ is the non-integer order derivative of a signal and it possesses non-local property. Fractional-order difference curvature (fDC) is a new feature descriptor and it is very useful for differentiating textures, ramps and flat regions [2]. In order to identify the edges, ramps, and flat areas, it is proposed to apply both fractional-order gradient and fractional-order difference curvature.

3.1 A New Diffusion Coefficient

The new coefficient of diffusion is defined as

$$C(|D^\alpha u|, fDC) = \frac{1}{1 + |D^\alpha u| + fDC} \quad (13)$$

Where, $|D^\alpha u|$ is the magnitude of fractional-order gradient and it can be calculated as $\sqrt{(D_x^\alpha u(x, y))^2 + (D_y^\alpha u(x, y))^2}$

The fractional-order difference curvature can be described by the formula

$$fDC = \left||u_{\eta\eta}^\beta| - |u_{\xi\xi}^\beta|\right| \quad (14)$$

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Research Article | Volume 10, Issue 4 | Pages 837-842 | e-ISSN: 2347-470X

Where $u^{\beta}_{\eta\eta}$ and $u^{\beta}_{\xi\xi}$ correspondingly denote the fractional-order derivatives in the level curves' perpendicular and tangent directions, β represents fractional-order, and |.| denotes the absolute function.

$$u^{\beta}_{\eta\eta} = \left(\frac{D^{\beta}u}{|D^{\beta}u|}\right)^{T} \begin{pmatrix} D^{\beta}_{xx}u & D^{\beta}_{xy}u \\ D^{\beta}_{xy}u & D^{\beta}_{yy}u \end{pmatrix} \left(\frac{D^{\beta}u}{|D^{\beta}u|}\right) \quad (15)$$

$$= \frac{\left(D^{\beta}_{x}u\right)^{2} D^{\beta}_{xx}u + 2D^{\beta}_{x}u.D^{\beta}_{y}u.D^{\beta}_{xy}u + \left(D^{\beta}_{y}u\right)^{2} D^{\beta}_{yy}u}{(D^{\beta}_{x}u)^{2} + (D^{\beta}_{y}u)^{2}} \quad (16)$$

$$u^{\beta}_{\xi\xi} = \left(\frac{D^{\beta}u}{|D^{\beta}u|}\right)^{T} \begin{pmatrix} D^{\beta}_{xx}u & -D^{\beta}_{xy}u \\ -D^{\beta}_{xy}u & D^{\beta}_{yy}u \end{pmatrix} \left(\frac{D^{\beta}u}{|D^{\beta}u|}\right) \quad (17)$$

$$= \frac{\left(D^{\beta}_{x}u\right)^{2} D^{\beta}_{yy}u - 2D^{\beta}_{x}u.D^{\beta}_{y}u.D^{\beta}_{xy}u + \left(D^{\beta}_{y}u\right)^{2} D^{\beta}_{xx}u}{(D^{\beta}_{x}u)^{2} + (D^{\beta}_{y}u)^{2}} \quad (18)$$

Table 1 shows the study of the curvature operators in the three distinct regions of the degraded image.

Table 1: Comparison of Gradient and curvatures

Feature	Gradient	MC	GC	DC	fDC
Edge	Large	Large	Small	Large	Large
Flat/Ramp	Small	Small	Small	Small	Small
Isolated Noise	Large	Large	Large	Small	Small

The difference curvature (DC) and fractional-difference curvature (fDC) are thus observed to be minimal in homogeneous and noisy regions and only large on edge features. Naturally, the values of DC and fDC can be used to identify edges from noisy and flat areas. However, the fDC will give the thin edge output [2]. Therefore, the following is the proposed fractional-order image denoising model based on the fractional-order gradient and fractional-order difference curvature:

$$\frac{\partial u}{\partial t} = -D^{\alpha}(C(|D^{\alpha}u|, fDC)D^{\alpha}u) \quad (19)$$

The above equation can be represented as

$$\frac{\partial u}{\partial t} = -D^{\alpha}_{x}(C(|D^{\alpha}u|, fDC)D^{\alpha}_{x}u) - D^{\alpha}_{y}\left(C(|D^{\alpha}u|, fDC)D^{\alpha}_{y}u\right) \quad (20)$$

The performance of the proposed image denoising model is explained as follows. For noisy pixels, a small fDC value is chosen which will accelerate the diffusion process of noisy pixels. Therefore, the proposed model can effectively eliminate the noise and preserve more details. The value of fDC for edges is very high. As a result, the coefficient of diffusion value has a tendency to be very low, which will cause the diffusion of edges to go more slowly and preserve the properties of the image. Therefore, the proposed methodology can change the diffusion speed of edges, features, and noise in the damaged image. This approach has a strong capability of removing noise also preserving edge details. The following algorithm provides a summary of the suggested model's overall restoration procedure.

1. **Initialization:**
   1.1 $u^{(0)} = u,\ \Delta t = 0.01,\ \alpha, \beta$
   1.2 Calculate the discrete Fourier transform $\hat{u}^{(0)}$ of $u^{(0)}$.
2. **Iteration:** For $i = 0,1,2,3, \ldots$; Calculate $\hat{u}^{(i+1)}$ by the following steps

2.1 Calculate the fractional-order central partial differences $\widetilde{D}^{\alpha}_{x}u^{(i)}, \widetilde{D}^{\alpha}_{y}u^{(i)}$using (9) and (10).

2.2 Calculate $\left|\widetilde{D}^{\alpha}u^{(m)}\right| = \sqrt{\left(\widetilde{D}^{\alpha}_{x}u^{(i)}\right)^{2} + \left(\widetilde{D}^{\alpha}_{y}u^{(i)}\right)^{2} + \epsilon}$, ϵ is a small value to avoid divide by zero.

2.3 Compute fractional-order difference curvature fDC using Eq. (14)

2.4 Compute conductance coefficient $C(|D^{\alpha}u|, fDC)$ using Eq. (11)

2.5 Calculate $g^{(i)}_{x} = C(|D^{\alpha}u|, fDC)\widetilde{D}^{\alpha}_{x}u^{(i)}$ and $g^{(i)}_{y} = C(|D^{\alpha}u|, fDC)\widetilde{D}^{\alpha}_{y}u^{(i)}$

2.6 Calculate $\hat{g}^{(i)} = [K^{*}_{1}][F(g^{(i)}_{x})] + [K^{*}_{2}][F(g^{(i)}_{y})]$

2.7 Calculate $\hat{u}^{(i+1)} = \hat{u}^{(i)} - \hat{g}^{(i)}\,\Delta t$

2.8 Calculate inverse discrete Fourier transform of $\hat{u}^{(i+1)}$i.e., $u^{(i+1)}$

2.9 If PSNR $(u^{(i+1)}, u_0)$ > PSNR $(u^{(i)}, u_0)$, then set $i = i + 1$; and go to step 2; else stop.

4. NUMERICAL EXPERIMENTS

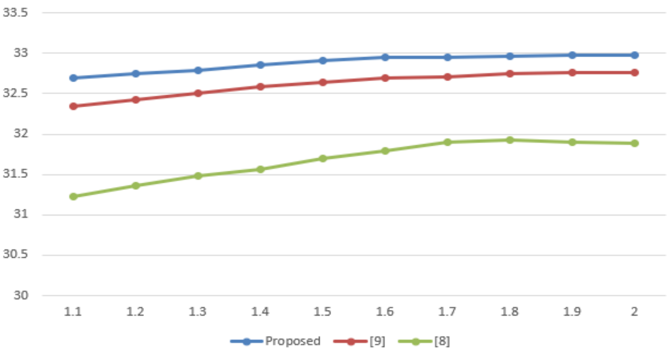
The USC-SIPI image database is used to denoise the images for this study. The original images are degraded by Gaussian noise with a mean value of zero and standard deviations of σ=10%, σ=20%, and σ=30% to evaluate the suggested framework's performance. In the first experiment, we considered the images which are damaged with Gaussian noise of mean equal to zero and σ=10%. The proposed algorithm is tested for the various values of fractional-order gradient in the range [8,14] of step size 0.1 and applied on Gaussian noisy boat image with 10% standard deviation. This algorithm is compared with the model [7] and the model [6] for the selection of the fractional-order. The comparison results are shown in the *figure 1.* It is observed that the proposed model gives better results when the fractional order is 1.9 and $\beta = 1.1$.

The comparable results are shown in *table 2* in terms of PSNR, MSSIM [15], and FoM [16] for a variety of images with Gaussian noise degradation and average value of zero and standard deviations of 10%, 20%, and 30%. It is noted that the suggested model works better than the other models since the diffusion coefficient is based on the fractional-order difference curvature. Hence thin edges are also preserved. As the noise density increases, the proposed model can able to remove the

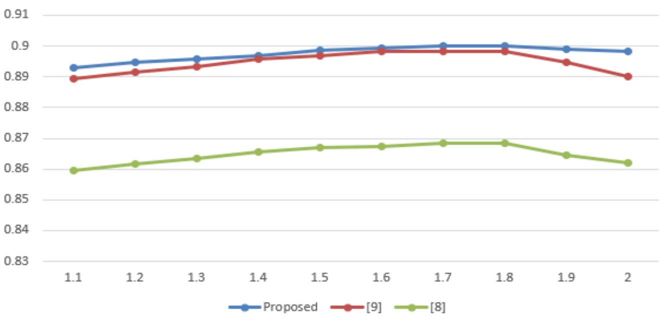
Research Article | Volume 10, Issue 4 | Pages 837-842 | e-ISSN: 2347-470X

noise effectively. The proposed model is applied on the damaged boat image of zero mean, 30% of standard deviation Gaussian noise and the results are presented in *figure 2*.

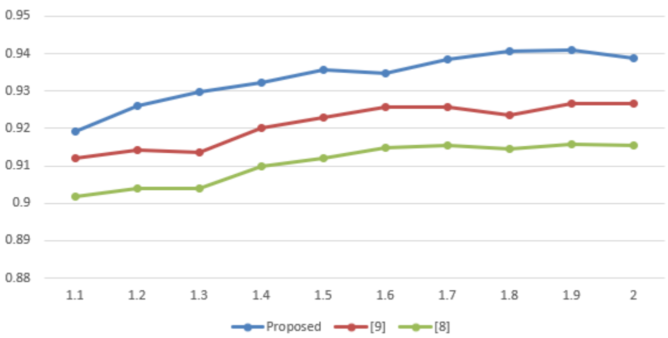
Comparisons are made between the suggested model and the models [7], [6], and [1]. The model [7] preserves edges textures well in denoising process. When the fractional order is 1.9 the model produces good results. When the noise density increases, i.e., more than 20% this model will not produce satisfactory results. The conduction coefficient in the proposed model depends on both fractional-order gradient and fractional-order difference curvature hence it can diffuse slowly at the edges and texture regions also more diffusion in the smooth regions. The figure of merit is more and clearly indicates that it can able to preserve the edges. As a result, the suggested model can successfully eliminate noise while preserving the edges and texture regions.



(a)



(b)



(c)

Figure 1: Selection of the fractional-order about boat image with Gaussian noise, σ=10% (a) PSNR (dB) (b) MSSIM (c) FoM

5. CONCLUSION

The proposed nonlinear diffusion model for image denoising reported in this study is driven by fractional-order gradient and fractional-order differential curvature. Fractional-order gradients have non-local property and can discern between texture, fine structures, and noise. Fractional-order difference curvature is an efficient edge descriptor even in the high-density noise. The selection of the order is more flexible for the fractional order gradient and fractional-order difference curvature; hence the model produces good quality results. The experimental results demonstrate that the proposed model decreases noise while avoiding visual distortions like the stair-case effect and speckle noise. Edge preservation is improved with respect to the state-of-art models. The selection of the fractional order is not constant for all the images. So, an adaptive denoising model may be proposed in the future work.

Table 2. Comparison of Gradient and curvatures

Image	σ	[7]	[6]	[1]	Proposed
Cameraman (256 × 256)	10%	31.0253	33.0216	32.2983	33.4006
		0.8054	0.8230	0.8209	0.8279
		0.9458	0.9408	0.9397	0.9512
	20%	28.0408	28.5250	28.4904	29.2138
		0.7180	0.7296	0.7359	0.7360
		0.7147	0.8899	0.7390	0.8884
	30%	21.9506	24.4120	26.6263	27.5758
		0.3017	0.4227	0.6805	0.6761
		0.6262	0.5354	0.5484	0.8471
Lena (512 ×512)	10%	32.6995	33.9139	33.7993	34.0364
		0.8624	0.8848	0.8848	0.8892
		0.8837	0.9117	0.8947	0.9274
	20%	30.5485	31.1852	30.4485	31.0134
		0.8129	0.8231	0.8129	0.8228
		0.7789	0.8373	0.8790	0.8730
	30%	26.0458	25.3835	28.7386	29.8377
		0.6852	0.4945	0.7595	0.7631
		0.7333	0.4141	0.7502	0.8298
Barbara (512 ×512)	10%	30.9425	31.6089	30.9456	31.5411
		0.7675	0.9038	0.8759	0.8959
		0.8626	0.9227	0.8726	0.9051
	20%	26.9230	28.1184	26.9303	27.8130
		0.7312	0.8114	0.7412	0.7814
		0.6581	0.8341	0.6681	0.8327
	30%	24.8092	24.3177	24.8109	26.0743
		0.6108	0.5704	0.6286	0.6784
		0.5166	0.5258	0.6176	0.7117
Boat (512 ×512)	10%	31.9221	32.7657	32.6275	32.9799
		0.8684	0.8984	0.8983	0.8999
		0.9157	0.9267	0.9302	0.9410
	20%	30.0630	28.2331	29.3319	30.1106
		0.7910	0.7199	0.7990	0.7963
		0.8178	0.8130	0.8302	0.8857
	30%	27.4345	27.2140	27.4097	28.6407
		0.7004	0.6978	0.7078	0.7179
		0.6705	0.7132	0.7325	0.8228
Peppers (512 ×512)	10%	32.1470	33.1352	32.6470	33.2661
		0.8667	0.8718	0.8679	0.8760
		0.8110	0.8888	0.8105	0.9157
	20%	29.5646	31.0158	29.6467	30.9396
		0.7789	0.8180	0.7896	0.8184
		0.6788	0.8004	0.6884	0.8471
	30%	27.8455	27.9797	27.8554	29.2368
		0.7337	0.7348	0.7379	0.7697
		0.5897	0.5858	0.5971	0.7877

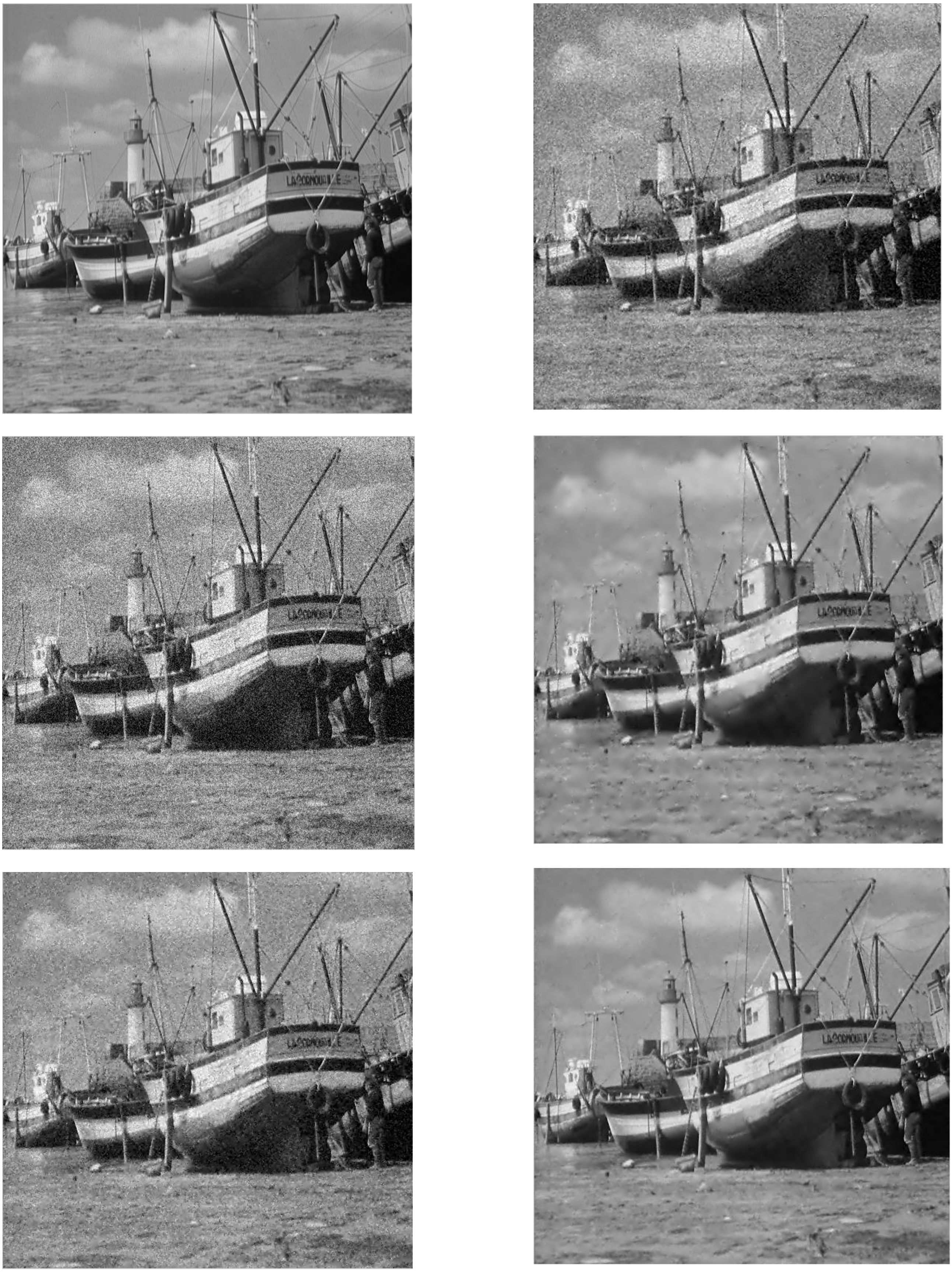


Figure 2: Comparison of various image denoising models about boat image with Gaussian noise, σ = 30%.
First Column: Original image, Noisy image, Result using [7];
Second Column: Result using [6], Result using [1], Result using proposed model

International Journal of Electrical and Electronics Research (IJEER)
Research Article | Volume 10, Issue 4 | Pages 837-842 | e-ISSN: 2347-470X

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