FOREX Publication
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International Journal of
Electrical and Electronics Research (IJEER)
Research Article | Volume 11, Issue 1 | Pages 112-118 | e-ISSN: 2347-470X

Array Design Using Genetic Algorithm for the Generation of Sum and Difference Patterns

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ABSTRACT- In the field of pattern synthesis, the design of arrays to synthesis optimized sum and difference patterns in a successive manner is an important problem in most of the radars. Although several researchers published many papers in the open literature, investigations are made by the authors to design arrays of discrete radiators using a well-formulated Genetic Algorithm (GA). The practical constraints are taken into account to design an amplitude distribution to produce the sum pattern with desired side lobe levels without broadening the main beam. Such sum patterns are found to be very useful in high-resolution radars where the EMI problems are also addressed.

In IFF radars, both sum and difference patterns are extremely useful to distinguish between friendly aircraft and enemy aircraft. Such patterns are conventionally produced involving two antennas. One is a directional antenna and another is an omnidirectional antenna. However, in the present work, they are generated using only one directional antenna. The patterns with a deep null in the boresight direction and difference lobes with a high slope are produced with an antiphase excitation to one-half of the array.

Keywords: Sum pattern, Difference pattern, Genetic Algorithm, Half Wave Dipole (HWD), Microstrip patch antenna.

ARTICLE INFORMATION
Author(s): Ramya K, Dr G S N Raju and Dr Sunny Dayal PA;
Received: 13/09/2022; **Accepted:** 12/02/2022; **Published**: 15/03/2023;
e-ISSN: 2347-470X;
Paper Id: IJEER220754;
Citation: 10.37391/IJEER.110115
Webpage-link:
https://ijeer.forexjournal.co.in/archive/volume-11/ijeer-110115.html

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1. INTRODUCTION

In today's modern applications, an antenna is the most important device to make communication links [20]. The non-isotropic radiators are dipoles, monopoles, waveguides, horns, slots, microstrips, etc. [1]. In this paper, dipoles and microstrip patch antennas are considered.

The dipole antenna is a very familiar practical antenna [18]. There are many types of dipole antennas, such are Hertzian dipole, short dipole, and half wave dipole [18]. Among these, the half-wave dipole is a widely used antenna due to its advantages [9]. If the length of the dipole is half of the wavelength, it is called a half-wave dipole [9]. Generally, each conductor length in the half-wave dipole is a quarter wavelength ($\lambda/4$) to make the total length $\lambda/2$ [21]. At present, half wave dipole antenna array is considered.
The dipole can be designed using wire or rod with a center-fed [21]. The dipole antenna structure is shown in *figure 1*. It consists of two wires or rods which are made up of metal conductors and these two are spaced with a gap between them to give feed to the antenna. Its geometric parameters are Length L, Feeding gap g, frequency, wavelength, and radiation resistance [17]. The radio frequency voltage is applied between the two poles (conductors) as feed, therefore the current flows through the conductors, which causes an electromagnetic signal to be radiated [21].

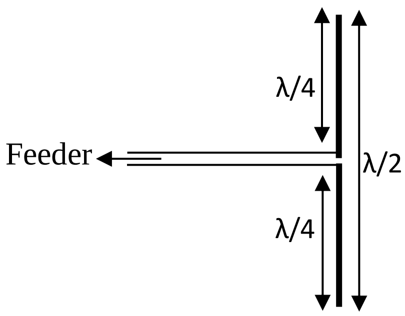


Figure 1: Half-wave dipole with center feed

Low cost and minimum size with high-performance antennas are required in modern communication systems to minimize the communication equipment size [2]. one such antenna is a microstrip patch which fulfills the requirement of present communication systems [2]. It operates under microwave frequency range, a so-called microstrip antenna. It can be fabricated onto any surface using a photolithography etching process [2]. Due to its high compactness and easy fitting, it is used in various applications like mobile and satellite communications, Global Positioning System applications, Radio Frequency Identification (RFID), Radar Applications, Telemedicine Applications [wearable antenna], etc. [22]. The microstrip patch antenna (MPA) consists of 3 layers, ground plane followed by a dielectric substrate on top of which radiating patch is fabricated [3] as shown in *figure 2*.

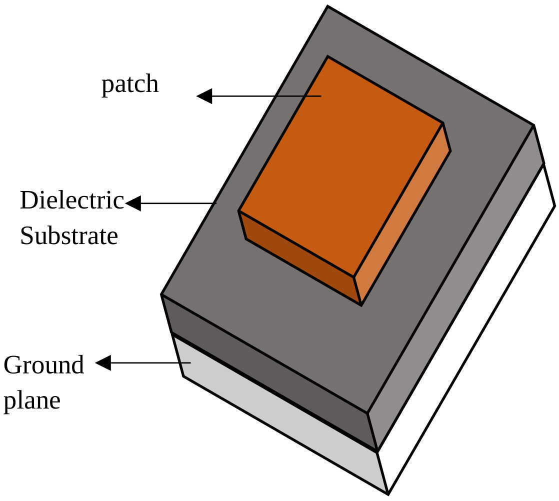
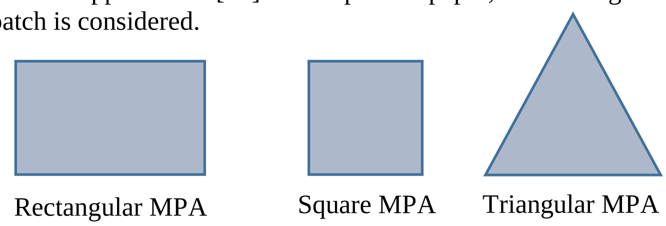


Figure 2: Microstrip patch antenna

Generally, the patch is made up of conducting material like copper or gold and it can be made in different shapes like square, rectangular, circular, triangular, elliptical, or some other general shapes [3] as shown in *figure 3*. Each shape has its own merits and demerits as well as different applications. For example, circularly polarized radiation patterns are required in satellite communication and these can be achieved using circular or square MPA [22]. square-shaped MPA can also be used in 5G smartphone applications [4]. Elliptical MPA is used in Wireless Local Area Networks (WLAN), Ultra-Wide Band (UWB) communications, Super Wide Band (SWB) communications, Wi-Max communication systems, and Microwave communications, etc. [6]. Triangular MPA can be used as a broadband radiator, dual frequency, and multiband antenna and also the equilateral triangular patches are used to form bowtie antenna for indoor communication [27]. The dipole patch substrate is small enough to place in a handset with low SAR, it can be used in a 4G wireless communication system [14]. the ring sector MPA can be used in WLAN applications [15]. The circular ring MPA is used in wide-band and ultra-wideband applications [7]. Among all the different shapes of MPA, the basic and commonly used shapes for most demandable applications are rectangular and circular patch shapes. The rectangular patch geometrics can be separable, it is simple to analyze, and it can be used in all microstrip patch antenna applications [22]. In the present paper, the rectangular patch is considered.



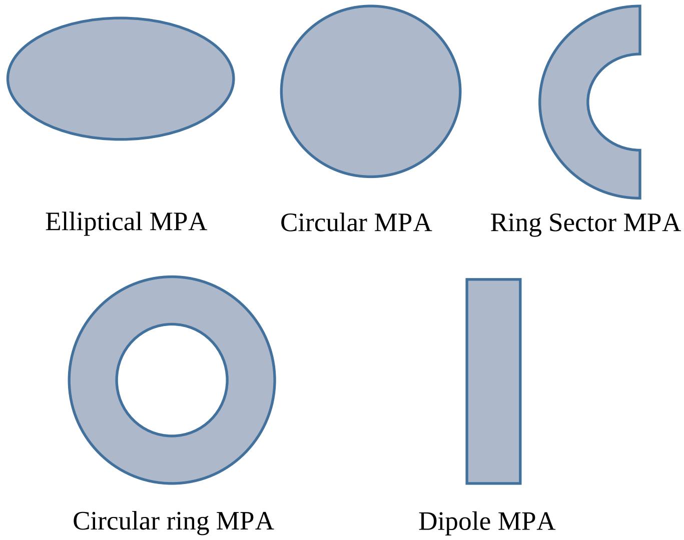


Figure 3: Common shapes of microstrip patch elements

The array of elements is required to achieve high gain, improved directivity and to solve impedance problems [10, 12]. The process of achieving the desired pattern of an array is called pattern synthesis [10, 12]. It can be attained by optimizing amplitude excitation or phase or both amplitude and phase or by adjusting space between the elements or any of these combinations [8, 11]. Many optimization techniques are being used by researchers to achieve high gain, minimum side lobe level, and desired pattern shape [13].

In this paper, a genetic algorithm is used to generate the amplitude weights for both half wave dipole array and microstrip patch array to achieve minimum side lobe level of sum patterns.

The sum pattern consists of the main lobe in the boresight direction and the number of minor lobes beside the main lobe called side lobes. For better performance, the main lobe width must be narrow and the side lobes level should be minimum [24, 28-30]. The difference pattern consists of null in the boresight direction and difference lobes have existed besides null.

2. GENETIC ALGORITHM

A genetic algorithm is chosen to determine the excitations of array elements. It is a kind of evolutionary search algorithm to give solutions to optimization problems based on natural selection [5]. It starts with initializing the population of possible solutions by natural random selection, solutions are nothing but chromosomes [16]. The array of genes is called chromosomes, genes are the basic building blocks of the genetic algorithm [23]. The chromosomes are selected according to their cost function and will be ranked from best fit to least fit [23]. The least-fit or unacceptable chromosomes are discarded and best-fit chromosomes are used to form the next generation [23]. After selecting the parent chromosomes, the children are produced by recombining and mutating the parent chromosomes by using the genetic operators called crossover and mutation [23]. The optimization process steps are as follows:

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1. Randomly generate the initial population, it should not be too small. It doesn't give sufficient room to search effectively and also it should not be too large because it couldn't expect a solution in a reasonable time [25]. The population size should be sufficient to cover the search space in less time.
2. Calculate the fitness function of each individual in the population.
3. Select the best-fit chromosomes as parents from the total population based on their fitness calculation [19].
4. Crossover (recombine) the selected parent chromosomes to produce new children.
5. Mutate the chromosomes, mutation is the process to make small random changes in chromosomes [23].
6. Check the requirement criteria, if it met the stopping criteria, stop the algorithm and return the best-fitted children.
7. Otherwise, go to step 3.

The Genetic Algorithm is one of the best optimization techniques to solve complex and nonlinear problems, but in the algorithm, there is an element of randomness with which, the computations time will become slightly higher for very large arrays.

3. FORMULATION

The resultant radiation pattern of a non-isotropic element array is the multiplication of the element pattern and array factor [1]. Consider the array of N non-isotropic radiating elements as shown in *figure 4* which are identical and equally spaced from -1 to 1 using the Ishimaru spacing formula.

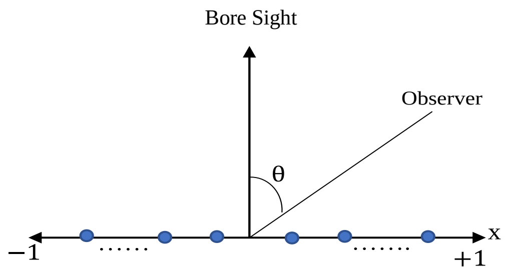

Figure 4: Geometry of N element symmetric linear array

The resultant field equation of N element linear array is given by [1]

$$E(\theta,\Phi) = F(\theta,\Phi)\sum_{n=1}^{N} A(x_n)e^{j\frac{2\pi}{\lambda}L[ux_n+\Phi(x_n)]} \quad (1)$$

Where $F(\theta,\Phi)$ = element field pattern,
$A(x_n)$ = Amplitude excitation weights of the nth element
x_n= Position of n^{th} element in the array.
$2L/\lambda$ = Normalized length of array
λ = Wave length, L = Length of the array
$u = \sin\theta$
θ = angle of the line of the observer with the broadside
$\Phi(x_n)$ = Phase excitation of the nth element

In the present work, $\Phi(x_n) = 0$ as Amplitude only control is considered.

3.1 Difference Pattern

The difference pattern is a radiation pattern that contains a null in the boresight direction. On either side of the null, there will be two major lobes of equal height in symmetric arrays. It can be generated from the discrete array equation shown in equation (1) by giving 180 degrees phase shift in one-half of the array [19]. The equation to generate the difference pattern for an N-element array is given as

$$E_d(u) = F(\theta,\Phi)\left[\sum_{n=1}^{\frac{N}{2}} A(x_n)e^{j(2\pi L/\lambda[ux_n+\Phi(x_n)]+\alpha)}dx + \sum_{n=\frac{N}{2}}^{N} A(x_n)e^{j((2\pi L/\lambda[ux_n+\Phi(x_n)]+\alpha))}dx\right] \quad (2)$$

Where α is additional phase to produce the difference patterns in order to produce null in the bore sight direction. α should be 'π' for one half of the array of elements and it is '0' for the second half then the equation (2) becomes (3).

$$E_d(u) = F(\theta,\Phi)\left[\sum_{n=1}^{\frac{N}{2}} A(x_n)e^{j(2\pi L/\lambda[ux_n+\Phi(x_n)]+\pi)}dx + \sum_{n=\frac{N}{2}}^{N} A(x_n)e^{j((2\pi L/\lambda[ux_n+\Phi(x_n)]+0))}dx\right] \quad (3)$$

3.2 Element Pattern of Half-Wave Dipole

The center feed halfwave dipole, which is a linear and cylindrical conductor placed along the Z-axis of the coordinate system is shown in *figure 5*. The number of these dipoles is used as array elements and arranged with Ishimaru spacing.

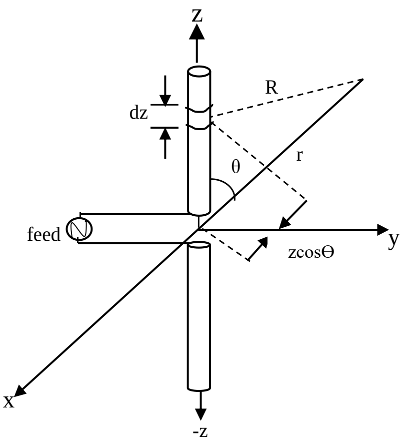

Figure 5: Typical geometry of dipole antenna

The E_θ component of half wave dipole element can be derived from *figure 5*, it is nothing but electric field strength, it is given by

$$F(\theta,\Phi) = E_\theta = \frac{j\eta I_m e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right] \quad (4)$$

Here $\eta = 120\pi$

I_m = maximum current distribution of the element

β = wave number = $\frac{2\pi}{\lambda}$

r = Approximated distance from dipole element to the observer

3.2 Element Pattern of Microstrip Patch Antenna

The electric field components of microstrip patch antenna are given by

$$E_\theta = \frac{\sin\left(\frac{KW\sin\theta\sin\Phi}{2}\right)}{\left(\frac{KW\sin\theta\sin\Phi}{2}\right)}\cos\left(\frac{KL}{2}\sin\theta\cos\Phi\right)\cos\Phi \qquad (5)$$

$$E_\Phi = \frac{\sin\left(\frac{KW\sin\theta\sin\Phi}{2}\right)}{\left(\frac{KW\sin\theta\sin\Phi}{2}\right)}\cos\left(\frac{KL}{2}\sin\theta\cos\Phi\right)\cos\theta\sin\Phi \qquad (6)$$

Where K = wave number = $\frac{2\pi}{\lambda}$

W = Width of the rectangular microstrip patch

L = length of the rectangular microstrip patch

The magnitude of the fields is given by

$$F(\theta,\Phi) = \sqrt{E_\theta^2 + E_\Phi^2} \qquad (7)$$

4. RESULTS AND DISCUSSION

Computations are carried out for the design of amplitude distributions using MATLAB programming language to implement GA for the arrays of different elements to obtain the specified radiation patterns. The desired SLL is set at -30 dB, to achieve this cost function, the random values are assigned to the genetic parameters like population size, mutation rate, maximum generations, and maximum function. The algorithm will be stopped if the desired SLL is achieved for the given random values. For example, when 40 element microstrip array is chosen, the given random values of genetic parameters to achieve -30dB SLL are as follows, population size is 300, the mutation rate is 0.01, the maximum number of generations is 1200 and the maximum function calls are 1100000. Similarly, for different types and the number of array elements, the random values are chosen until achieving the desired cost function. For arrays containing the number of elements equal to 40 and 80, the results of amplitude distributions are presented in *figures 6 to 9*.

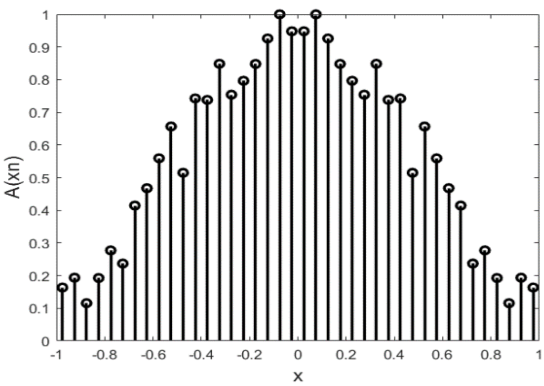

Figure 6: Normalized amplitude distribution for N = 40 Half wave dipole Elements

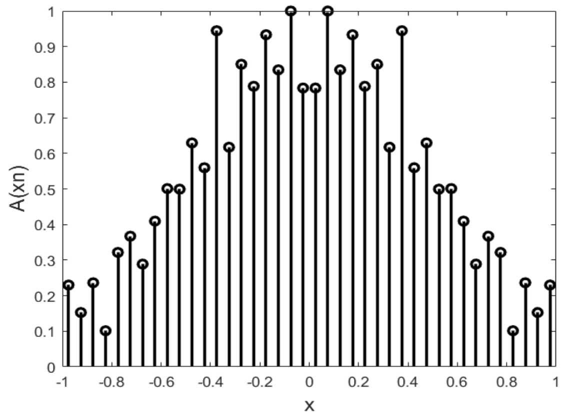

Figure 7: Normalized amplitude distribution for N = 40 Microstrip Patch antenna Elements

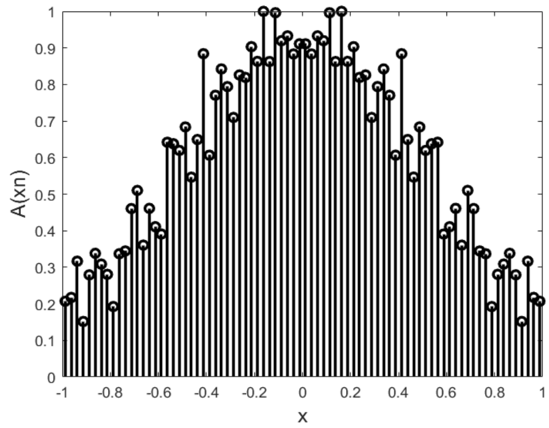

Figure 8: Normalized amplitude distribution for N = 80 Half wave dipole Elements

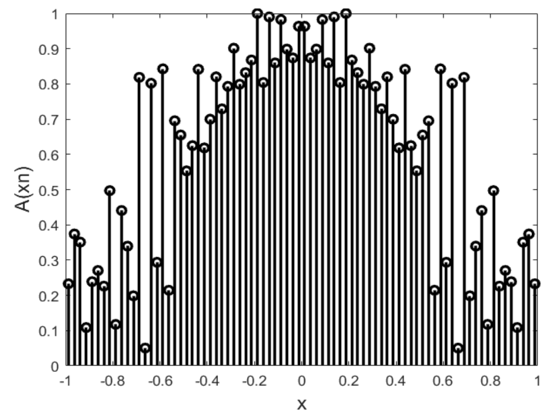

Figure 9: Normalized amplitude distribution for N = 80 Rectangular Microstrip Patch Elements

The amplitude distributions presented are obtained from continuous distributions by taking the discrete levels by sampling at the locations of the elements. The element locations are computed using the Ishimaru spacing function. It is evident from the results; the amplitude distributions are symmetric and partially tapered toward the end of the array. The patterns are presented only in the visible region i.e., from θ is -90 to +90. However, the normalized radiation patterns are presented as a function of sinθ (u). u varies from -1 to +1.

As isotropic radiators are ideal, practical elements like dipoles and microstrip antennas are considered. Using the amplitude distributions so obtained are applied to the arrays of the elements mentioned above and radiation patterns are numerically computed. In addition to the amplitude distribution, antiphase excitation to one-half of the array is applied and patterns are recomputed. Amplitude-only control patterns are nothing but sum patterns and phase-added patterns are different patterns. The generation of sum and difference patterns involves mathematical equations as mentioned, these equations are implemented using MATLAB tool. The results on sum and difference patterns for the above arrays are presented in *figures 10 to 15.*

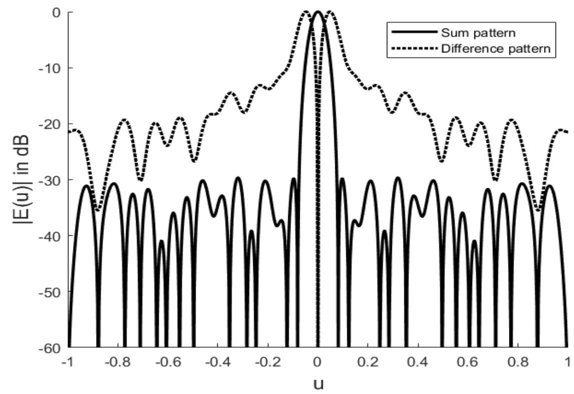

Figure 10: Normalized Sum and SLS Difference pattern with amplitude only control for N = 40 Half wave dipole Elements

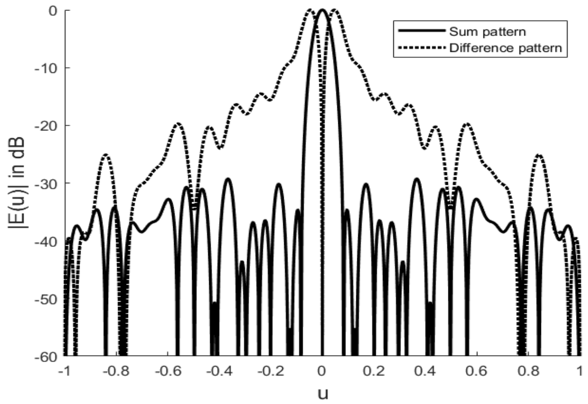

Figure 11: Normalized Sum and SLS Difference pattern with amplitude-only control for N = 40 Microstrip patch Elements

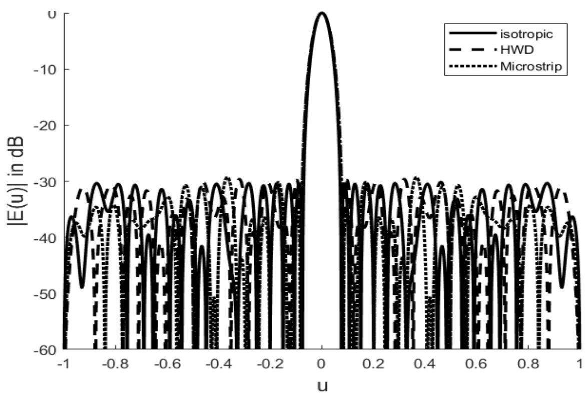

Figure 12: Sum pattern comparison with amplitude-only control for N = 40 Elements

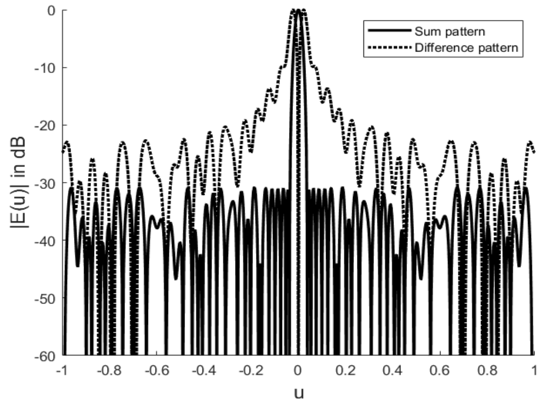

Figure 13: Normalized Sum and SLS Difference pattern with amplitude only control for N = 80 Half wave dipole Elements

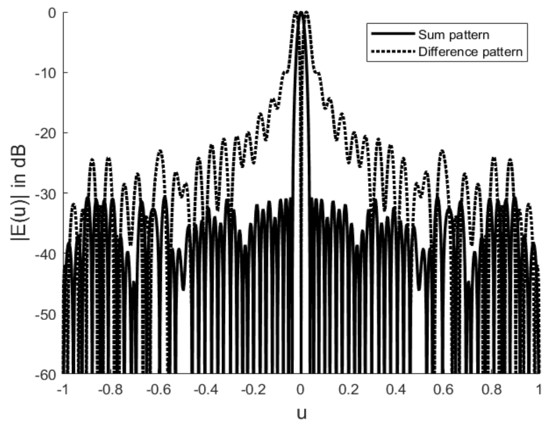

Figure 14: Normalized Sum and SLS Difference pattern with amplitude-only control for N = 80 Microstrip patch Elements

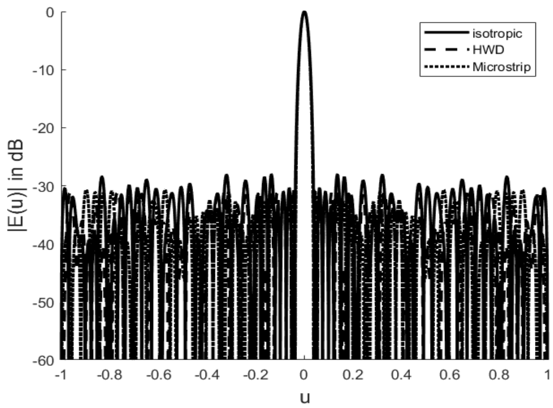

Figure 15: Sum pattern comparison with amplitude only control for N = 80 Elements

In all the presented patterns, sum and difference patterns are overlapped. It is evident from all the patterns; the difference null is deep and the difference lobe is high. Moreover, the difference lobes in all the cases are much Above the side lobe levels and hence side load suppression takes place. It is also found from the patterns of the present work the main beam width of the sum pattern is well-controlled without raising the side lobe levels. This is due to the constraints made on the side lobe levels in the design itself. The present design is also found to be very useful as the overall pattern structures are not disturbed even after practical elements are incorporated in the arrays.

The present design is valid both for small and large arrays. The results of the present work are also extremely useful for all the frequency ranges as the design is made independent of frequency. The simulations are carried out and the results are compared and validated with those of practical elements in the arrays.

5. CONCLUSION

The sum and difference patterns designed in the present work are found to be well controlled by the amplitude and phase distributions. The side lobe levels are restricted to -30 dB in the design. The desired SLL is achieved as the levels are defined in the constraints of GA itself. The difference patterns are found to exhibit high difference slopes with a deep null in the boresight and the difference slopes are at a level more than 8dB above that of side lobe sum patterns.

In the open literature, in particular, the Skolnik radar handbook [26], reported two antenna patterns where the difference lobes have very small slopes and the null in the boresight is not that deep when compared to the present patterns. The comparative studies are also made between the patterns of arrays of discrete radiators and those of arrays of practical elements. The comparison is made by overlapping the patterns of both for the arrays of the same number of elements. The results are self-explanatory and are prone to be more useful in IFF applications and also useful in scanning applications than those of two antenna patterns as quoted in the reference [26].

It has been possible to achieve feasible results as a single antenna array is considered to generate both sum and difference patterns by controlling excitation only. The sum pattern has distinct structures depending on the arrays, with large arrays, the width of the main beam is smaller compared to those of small arrays and also the number of side lobes is increased with the number of elements. The investigations to be made in the future will be on further restricting the side lobe levels and on arrays of other practical elements.

6. ACKNOWLEDGMENTS

The authors express their gratitude to prof. Mukti Mishra, president and prof. DN Rao, Vice president of centurion university to support the present work.

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