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# Design and Analysis of Analog Feedback Communication System with Rayleigh Fading Channel Model

**Richa Tengshe[1*] and Navin Kumar[2]**

[1]*CMR Institute of Technology, Bengaluru, India, richatengshe@gmail.com*
[2]*Amrita School of Engineering Bengaluru, Amrita Vishwa Vidyapeetham, India, navinkumar@ieee.org*

***Correspondence:** Richa Tengshe; richatengshe@gmail.com

**ABSTRACT-** In this work, we mathematically model a wireless Analog feedback communication system (AFCS) using a Rayleigh fading channel. AFCS system is a new research area and has promising applications, especially in low-power devices such as sensors. Compared to AWGN, Rayleigh fading channel more closely models the real wireless environment. In this work, AFCS Rayleigh fading channel is considered in forward transmission while AWGN is considered in the feedback channel. We evaluated the performance of the system which is mostly based on the minimization of mean square error (MSE) at the receiver. Even in presence of a wireless fading environment, the AFCS attains a 0 MSE value in $2-3$ iterations if a noise power of $10dB$ is considered while in $10-15$ iterations if a noise power of $20dB$ is considered.

**Keywords:** Analog Feedback Communication System, ergodic capacity, mean square error, outage capacity, Rayleigh fading channel.

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## 1. INTRODUCTION

Wireless communication systems have evolved in the last few decades [1, 2]. Most of the modern communication systems use digital processing due to many benefits such as ease of design and modularity. However, Analog communication which was the area of investigation before 1970 has set many important milestones. In 1948, Shannon published a ground-breaking paper [3] that helped establish the field of information theory. Shannon calculated the theoretical upper and lower bounds for a reliable transmission for a specific source-channel pair using the concepts of entropy, information, and capacity. Shannon demonstrated that information can be transmitted with an arbitrarily small error if the rate at which information is transmitted is less than or equal to the maximum capacity of the channel. However, any specific coding techniques to achieve this were not discussed. Practical systems are not able to achieve this upper bound for data rate as it requires infinite block length codes. Since Shannon's ground-breaking work, numerous studies have been carried out to reach the theoretical limits, and researchers are still working to enhance performance and create new methods to produce better outcomes. Among these methods strong vector quantization techniques [4] and long capacity approaching codes like turbo codes [5, 6] and low-density parity-check codes (LDPC) [7, 8] strive to achieve the theoretical limits. However, reaching Shannon capacity boundary without the use of complex coding techniques and long block length codes is still elusive. Nevertheless, in the 1950-time frame, P. Elias [9] proved that the Analog feedback communication system (AFCS) could transmit the data at information boundaries without coding. This research work inspired research communities like [10], [11] whose work showed the possibility of AFCS that performs at the limit of the information. This was achieved by the statistical matching of source and channel pair. For example, it was shown that a Gaussian source can be transmitted optimally on an AWGN channel without complex coding. The extensive work done in [12], emphasizes the advantages of the feedback. Even though feedback cannot increase the capacity of the channel, it can reduce the coding/decoding complexity of the transmission-reception algorithm. These works were based on the direct mapping of source symbols to channel symbols. This method of communication is known as the joint source and channel coding (JSCC) technique, as opposed to the separation principle based digital communication systems, where the source and channel coding is performed independently.

Analog JSCC (AJSCC) and AFCS are streams of research that started in the 1990s. Both areas suggest the transmission of discrete time continuous amplitude samples which are mapped directly to channel symbols for the transmission. AJSCC technique suggests the transmission using spiral mapping curves. This technique finds its origin in individual works by Shannon [13] and Kotel'nikov [14], where the communication process was depicted geometrically. Following this in the previous decade, direct transmission using geometrical mappings was shown to be optimal and less complex by Chung in [15] and [16]. These mappings were named Shannon-Kotel'nikov mappings. Later, in [17] author proposed

Archimedes' spiral for AJSCC for AWGN channels, and [18] worked on a wireless channel model for AJSCC. AJSCC has transformed from the geometrical mapping suggested by Shannon and Kotel'nikov to deep learning and auto encoder based [19] studies and implementations. In three decades almost all performance enhancing techniques like diversity, MIMO, OFDM, and almost all sorts of channels like multiple access, broadcast, etc. have been explored in the field of AJSCC. Currently, a machine learning/deep learning-based approach is being investigated.

Another parallel research area recently being focused on is AFCS. This also has the same potential as AJSCC. This transmission technique is characterized by adaptively modulated symbols on the forward channel and the usage of a feedback channel for iteratively correcting the received symbols while minimizing the mean square error (MSE) of the estimation.

In this paper, we follow the approach of AFCS. In AFCS based system, one of the most important performance parameters considered is MSE at the receiver rather than bit error rate [20]. Minimizing MSE leads to reaching close to Shannon capacity boundary [21]. Minimum MSE is obtained using an iterative process via feedback samples. That is, a strong signal channel is assumed as a feedback channel in this type of system. Such a system is also characterized as a power efficient, less complex system and is expected to attract usage in low-powered sensors and networks. In [20] author discusses the application of AFCS to sensor node-base station physical link of the wireless sensor network (WSN). The author discussed the challenges of optimal utilization of resources due to interrelated performance criteria and highlights the sub-optimal digital links between the sensor node and base station. The author proposes the transmission of Analog signals using adaptive pulse amplitude modulation (PAM) which is adjusted over the feedback channel without any need for complex coding and digitizing. In another application-oriented research [22], the author proposes the use of AFCS for narrow-band satellites transmitting to the ground station. It is demonstrated that the use of high-quality feedback in the link from the ground station to the satellite improves transmission quality and spectral-energy efficiency over the forward link (from satellite to ground station). This also simplifies the forward transmitter architecture present in the satellite and reduces energy consumption.

Many studies on AFCS as a transmission-reception algorithm are discussed by the author for example [23, 24]. In these studies, the mean square error is the only performance criterion for optimization. In most of the studies, they used linear modulators and saturation through feedback. However, recently, authors propose a nonlinear model for the transmitter along with statistical fitting of the modulator to overcome the abnormal errors due to modulator saturation [25, 21]. Most of these studies on AFCS, consider AWGN channel in both directions. In this work, we developed the channel model for Rayleigh fading, especially in the forward direction assuming the existence of a noise free AWGN channel in the feedback path. Few applications like sensor networks are designed in line with this assumption. It is observed that even in Rayleigh fading environment, zero MSE can be obtained in fewer iterations of feedback samples. However, this convergence depends on the signal-to-noise ratio (SNR) of the forward and feedback channels. In [26] author has implemented receive diversity with maximum ratio combining technique to improve the performance of AFCS in a wireless environment. The main contributions to this work are listed below:

- Architecture of AFCS in wireless channel setup where the channel distribution is considered as Rayleigh fading.
- Carry out the performance evaluation of AFCS in the Rayleigh fading channel in terms of MSE per iteration for varying forward channel noise and feedback channel noise.
- Derived the expression for the threshold number of cycles in terms of forward and feedback channel noise for AFCS in Rayleigh fading.
- Derived the expression for Ergodic capacity, and outage probability and compared with AWGN AFCS capacity.

These contributions are covered in different sections further in this paper. *Section 2* elaborates the architecture of AFCS in the Rayleigh fading channel. *Section 3* covers the optimization algorithm and performance evaluation of the system. The simulation results are discussed in *section 4* and the conclusion in *section 5*.

## 2. SYSTEM ARCHITECTURE AND ALGORITHM

Analog transmission of a memory-less, discrete-time, continuous-amplitude Gaussian source band limited to $F\ Hz$ through a wireless channel is considered here. The source signal is assumed to have a mean $x_0$ and variance $\sigma_0^2$. The block diagram of the proposed AFCS system is shown in *Figure 1*. The system comprises transmitter-receiver pair in each forward and feedback path. T1-R1 is the forward channel transmitter-receiver pair whereas T2-R2 forms the feedback channel transmitter-receiver pair. The forward channel is assumed to be a Rayleigh fading flat channel while the AWGN channel of high quality with low noise is assumed to be the feedback channel. An unconstrained transmitter T2 transmitting from the receiver on the reverse channel ensures a good quality noiseless feedback channel. Such a scenario is very common in wireless sensor networks where the sensor node transmits Analog signals to the base station. Usually, the sensor nodes are power, and resource constrained but the base station may be able to afford transmission at a high-power level to ensure noiseless feedback.

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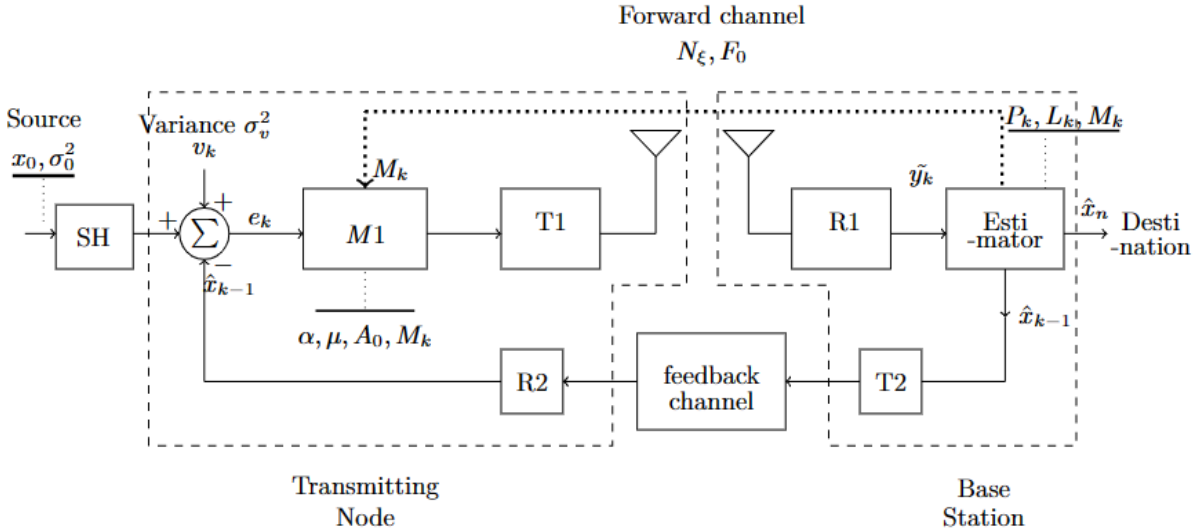


**Figure 1:** AFCS architecture with Rayleigh Fading Channel

In the forward path of the proposed architecture, the transmitting node has a source, a sampling and buffer circuit (SH), a comparator, a modulator (M1), and a radio frequency transmitting module (T1). The base station on the other end has a receiver (R1), a de-modulator (part of receiver R1), and an estimator in the forward path. In the feedback path, the base station consists of a modulator and radio frequency front-end transmitting system (T2), and the transmitting node consists of receiver R2.

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The source sample is held at one of the inputs of the comparator during this time$T$. In each kth cycle$(k = 1,2, \ldots, n)$, the base station analysis the signal received from the transmitting node and computes the sample's intermediate estimate based on the previous estimate and the current observation. This estimate is preserved until the next cycle. The current estimate is sent back to the transmitting node over the feedback channel. This estimate appears at the second input of the comparator, and the error or difference between the actual and the estimate is calculated. This difference is modulated during the next cycle and is transmitted. The convergence gain $(L_k)$ of the estimator is optimized to reduce the mean square error between the actual input signal and the estimate to a minimum. During each cycle, the modulation index of the adaptive modulator is adjusted to a value that ensures that the abnormal errors due to rare events of over-modulation remain below a very small value(μ). Such a situation is known as the probability of modulator saturation or over-modulation.

The maximum number of cycle in which a single input sample can be transmitted is $n = T/(t_0) = F_0/F$, where $T = 1/(2F)$ is the sampling period and $t_0 = 1/(2F_0)$ is the length of one cycle of transmission (time duration for transmission of one sample). $F_0 = 1/(2t_0)$ is the minimum bandwidth needed for the forward channel, the feedback channel is assumed to have bandwidth more than $2F_0$.

The error signal at the comparator's output $e_k$ in the $k^{th}$ cycle of transmission of a sample of input, where $k = 1,2, \ldots n$ is given by:

$$e_k = x_k - \hat{x}_{k-1} + v_k \quad (1)$$

Here $v_k$ is considered an independent and identically distributed AWGN noise having variance $\sigma_v^2$ modelled to consider feedback errors into account. This error signal $e_k$ is modulated by the adaptive pulse amplitude modulator (PAM) whose transfer characteristic is given by:

$$s_k = A_0 \begin{cases} M_k e_k, & \text{if } M_k|e_k| \le 1 \\ sign(e_k), & \text{if } M_k|e_k| > 1 \end{cases} \quad (2)$$

The transfer characteristics in *equation (2)* considers a nonlinear characteristic for the modulator. This is done to take into account the errors appearing due to modulator saturation or over-modulation. Here $A_0$ is the carrier amplitude and $M_k$ is adaptively controlled modulation index of the modulator. Going forward the received signal after demodulation is represented by:

$$\tilde{s_k} = A_0 h \begin{cases} M_k e_k + \zeta_k, & \text{if } M_k|e_k| \le 1 \\ sign(e_k) + \zeta_k, & \text{if } M_k|e_k| > 1 \end{cases} \quad (3)$$

Here h is the Rayleigh fading coefficient and $\zeta_k$ is the independent and identically distributed Gaussian noise in forward channel with variance $\sigma_\zeta^2$. Noise in the feedback channel $(v_k)$, forward channel $(\zeta_k)$ and the input signal $(x_k)$ are assumed to be independent and uncorrelated. In each iteration, the probability of over-modulation of the adaptive modulator (AM), denoted as $\mathrm{Pr}_{\mathrm{over}}^{\mathrm{k}}$, is given by:

$$Pr_k^{over} = Pr(M_k e_k > 1 | \tilde{s}_1^{k-1}, \hat{x}_{k-1}, M_{k-1}) < \mu. \tag{4}$$

A set of parameters $\hat{x}_{k-1}, M_k$ which satisfy *equation (4)* forms an allowed set $\Omega_k$ which ensure that the condition $M_k|e_k| \leq 1$ is satisfied with a likelihood not less than $1-\mu$. Here $\mu$ is the likelihood that the modulator will get saturated and indicates the information loss due to the saturation of the modulator. As a result, the number $1-(1-\mu)^n \approx n$, which is similar to BER in digital communication systems, represents the frequency of mistakes induced due to over-modulation during transmission. Typically, $\mu$ values between $10^{-12} \leq \mu < 10^{-4}$ are effectively sufficient for the design. The parameters of the AM, $\hat{x}_{k-1}$ and $\mathrm{M_k}$, are determined using this value of $\mu$ and *equation (4)*. Such AM is known as statistically fitted AM. The equation for $M_k$ as proposed in [25] which takes into account statistical fitting condition is given by:

$$M_k = \frac{1}{\alpha\sqrt{P_{k-1}+\sigma_v^2}}. \tag{5}$$

Where $\alpha$ is the saturation factor which considers the chances of over modulation. The expression relating $\alpha$ and $\mu$ is given by:

$$\Phi(\alpha) = \frac{1}{\sqrt{2\pi}}\int_0^{\alpha} e^{\frac{-x^2}{2}}\, dx \geq \frac{1-\mu}{2}. \tag{6}$$

Here $\Phi(\alpha)$ is the well-known Gaussian error function. Following demodulation *equation (3)*, the signal is sent to the BS estimator. The estimator computes the current estimate of input $\hat{x}_k$ using the Kalman type equation given by:

$$\hat{x}_k = \hat{x}_{k-1} + L_k\tilde{s}_k\,, \tag{7}$$

This estimator *equation (7)* takes into account the contribution of the previous estimate and the current observation both. The convergence gain of the estimator $\mathrm{L_k}$ decides whether more weightage is given to the current observation or previous estimate. The value of $L_k$ governs how fast the estimate $\hat{x}_k$ converges to the original value $x$ of the transmitted sample. The optimization task computes the value of $L_k$ so that the mean square error is minimised. MSE in the $\mathrm{k^{th}}$ iteration denoted by $\mathrm{P_k}$ is given by:

$$P_k = E\{(x-\hat{x}_k)^2\}\,, \tag{8}$$

Where $E\{.\}$ stands for expectation and $P_{k-1}$ in *equation (5)* is the MSE in the $(k-1)^{th}$ cycle given by $E[(x-\hat{x}_{k-1})^2]$. MSE in *equation (8)* is affected by the estimation algorithm as well as by the AM parameters, $\hat{x}_{k-1}$, and $M_k$ at the transmitter. The values of $L_k$, $\hat{x}_{k-1}$, and $M_k$ is chosen to minimise MSE optimises both the transmission and reception algorithms.

Further, the current estimate is sent back to the transmission node through the feedback channel. The original sample is compared to the sample received over the feedback channel, and the deviation is relayed to the modulator. The modulation index $M_k$ is set to $M_{k+1}$ and the estimator calculates the convergence gain $L_{k+1}$ changes the value of $L_k$ to $L_{k+1}$, after which the $(k+1)^{th}$ cycle starts. The addressee receives the final estimate $\hat{x}_n$ of the input sample after $n$ cycles, and the next sample transmission begins. The saturation factor that establishes the tolerable level of transmitter saturation, the variance of the input signal $\sigma_0^2$ and the mean value $x_0$ of the input signal, respectively, determines the initial values of $\hat{x}_0$ and $M_1$

## 3. OPTIMIZING AFCS OVER RAYLEIGH FADING CHANNEL

The statistically fitted modulator mostly behaves as a linear unit, hence the non-linear transmitter model given by (2) can be swapped out for a linear one:

$$s_k = A_0 M_k e_k = A_0 M_k(x_k - \hat{x}_{k-1} + v_k). \tag{9}$$

This allows us to replace the model in (3) by (10),

$$\tilde{s}_k = A_0 h M_k(x_k - \hat{x}_{k-1} + v_k) + \zeta_k. \tag{10}$$

The unfitted ACS system {represented by *equation (2)*} and fitted ACS system {represented by *equation (3)*}, differs only with a small probability given by $n\mu$. As a result, the MSE of the two systems may only differ by $O(n)$. The statistical fitting, however, enables the transition of a non-linear optimization problem into a much simpler linear problem which can be handled using Bayesian estimation theory [21], [24], [25]. Now, as the fading coefficient is a complex quantity, the MSE $P_k$ takes the form:

$$P_k = E\{(x_k - \hat{x}_k)(x_k - \hat{x}_k)^*\}\,, \tag{11}$$

Where * denotes complex conjugation. Substituting *equation (7)* into *(11)* we have:

$$P_k = E\{(x_k - \hat{x}_{k-1} - L_k\tilde{s}_k)(x_k - \hat{x}_{k-1} - L_k\tilde{s}_k)^*\}\,, \tag{12}$$

$$P_k = E\{(x_k - \hat{x}_{k-1})(x_k - \hat{x}_{k-1})^* - L_k\tilde{s}_k(x_k - \hat{x}_{k-1}) - (x_k - \hat{x}_{k-1})L_k^*\tilde{s}_k^* + L_k\tilde{s}_k L_k^*\tilde{s}_k^*\}. \tag{13}$$

As we want to optimize $\mathrm{L_k}$ so that we get minimum MSE, we can differentiate *equation (13)* with respect to $L_k^*$, equate to zero and find the optimum expression for $L_k$ which minimizes the MSE

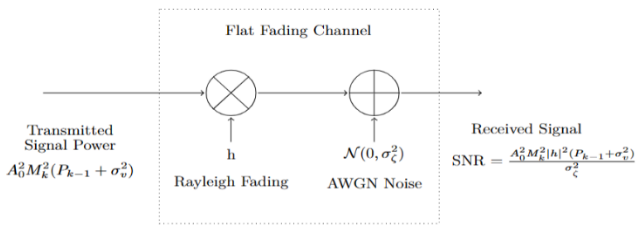


**Figure 2:** Rayleigh Fading Channel Model

$$\frac{\partial P_k}{\partial L_k^*} = -E\{(x_k - \hat{x}_{k-1})\tilde{s}_k^*\} + L_k E\{\tilde{s}_k\tilde{s}_k^*\} = 0, \tag{14}$$

$$E\{(x_k - \hat{x}_{k-1})\tilde{s}_k^*\} = L_k E\{\tilde{s}_k\tilde{s}_k^*\}\,, \tag{15}$$

$$L_k = \frac{E\{(x_k - \hat{x}_{k-1})\tilde{s}_k^*\}}{E\{\tilde{s}_k\tilde{s}_k^*\}}. \tag{16}$$

Now, substituting from *equation (10)* we get:

$$E\{(x_k-\hat{x}_{k-1})\tilde{s_k^*}\}=E\{(x_k-\hat{x}_{k-1})(hA_0M_k(x_k-\hat{x}_{k-1}+v_k)+\zeta_k)^*\}. \quad (17)$$

On simplification and considering $E[(x_k-\hat{x}_{k-1})]=0$, we get:

$$E\{(x_k-\hat{x}_{k-1})\tilde{s_k^*}\}=A_0M_kh^*E\{(x_k-\hat{x}_{k-1})(x_k-\hat{x}_{k-1})^*\}, \quad (18)$$

And as $P_{k-1}=E\{(x_k-\hat{x}_{k-1})(x_k-\hat{x}_{k-1})^*\}$ we get:

$$E\{(x_k-\hat{x}_{k-1})\tilde{s_k^*}\}=A_0M_kh^*P_{k-1}, \quad (19)$$

And

$$E\{\tilde{s_k}\tilde{s_k^*}\}=E\{\{hA_0M_k(x_k-\hat{x}_{k-1}+v_k)+\zeta_k\}\{hA_0M_k(x_k-\hat{x}_{k-1}+v_k)+\zeta_k\}^*\}, \quad (20)$$

As $hh^*=|h|^2$,$E\{v_kv_k^*\}=\sigma_v^2$ ,$E\{\zeta_k\zeta_k^*\}=\sigma_\zeta^2$ after simplification we get:

$$E\{\tilde{s_k}\tilde{s_k^*}\}=|h|^2A_0^2M_k^2(P_{k-1}+\sigma_v^2)+\sigma_\zeta^2. \quad (21)$$

Hence from *equations (16), (19) and (21)* $L_k$ is given by:

$$L_k=\frac{A_0M_kh^*P_{k-1}}{|h|^2A_0^2M_k^2(P_{k-1}+\sigma_v^2)+\sigma_\zeta^2}. \quad (22)$$

Substituting *equations (22) into (13)*, considering *equation (10)* and taking MSE in the prior cycle as:

$$P_{k-1}=E\{(x_k-\hat{x}_{k-1})(x_k-\hat{x}_{k-1})^*\}. \quad (23)$$

After simplification, we get:

$$P_k=\frac{\left[|h|^2A_0^2M_k^2\sigma_v^2+\sigma_\zeta^2\right]P_{k-1}}{|h|^2A_0^2M_k^2(P_{k-1}+\sigma_v^2)+\sigma_\zeta^2}, \quad (24)$$

From *equation (24)* we can infer that to achieve minimum MSE we have to search maximum value of $M_k$ from the permissible set of values which satisfy *equation (4).*

To summarize, the AFCS algorithm that achieves the lowest mean square error and ensures optimality is given by:

- Calculating the error signal, setting the permissible value for Mk in accordance with *equations (4) and (5),* modulating the error signal for transmission.
- Calculating the estimate of the transmitted signal using *equations (7) and (22).*
- Sending the estimate back to the transmitter over the feedback channel.
- Following this iterative algorithm for n cycles and delivering the estimate of nth cycle to the addressee.

## 3.1 Performance of AFCS on Rayleigh Fading Channel

Considering the transmitted signal as given in *equation (10)* the forward channel SNR $Q_h^2$ is given by:

$$Q_h^2=\frac{W_k^{sig}}{W^{noise}}=\frac{A_0^2M_k^2|h|^2(\sigma_v^2+P_{k-1})}{\sigma_\zeta^2}. \quad (25)$$

This is a random quantity and for a particular realization of $h$, the minimum MSE reachable in the $k_{th}$ cycle as given in *equation (24)*, considering *equation (25)* can be written as:

$$P_k=\frac{P_{k-1}}{1+Q_h^2}\left(1+\frac{Q_h^2\sigma_v^2}{\sigma_v^2+P_{k-1}}\right). \quad (26)$$

To analyze the dependence of MSE *equation (26)* on the number of cycles, we consider two cases. In the first case we consider $(1<k\le n^*)$ mean number of iterations/cycles upto $n^*$, where $n^*$ is the threshold number of cycles. When $(1<k\le n^*)$ $P_{k-1}/\sigma_v^2\gg 1+Q_h^2$ is valid hence from *equation (26)* we can see that MSE decreases exponentially as given by:

$$P_k=\sigma_0^2(1+Q_h^2)^{-k}. \quad (27)$$

Whereas after threshold number of cycles as $P_{k-1}$ becomes small $P_{k-1}/\sigma_v^2\ll 1+Q_h^2$ and the expression for $P_k$ becomes

$$P_k=\frac{\sigma_v^2}{k-n^*+1}. \quad (28)$$

The expression for threshold number of cycles $n^*$ is where $P_{n^*}$ becomes equal to $\sigma_v^2$. Hence, we can find $n^*$ from *equation (26)* by substituting $P_{n^*}=\sigma_v^2$ as:

$$n^*=\frac{1}{\log_2(1+Q_h^2)}\log_2\left(\frac{\sigma_0^2}{\sigma_v^2}\right). \quad (29)$$

### 3.1.1 Ergodic Capacity of AFCS

For a flat fading channel model as shown in *figure 2*, with exact channel information present (Channel state information (CSI)) at the receiver, the capacity of single input single output (SISO) forward channel is given by [27]:

$$C=\log_2(1+SNR)=\log_2\left(1+\frac{A_0^2M_k^2|h|^2(P_{k-1}+\sigma_v^2)}{\sigma_\zeta^2}\right), \quad (30)$$

Where bits/s/Hz is the unit of capacity. The $h$ is presumed to be flat and of block fading type. Even if $h$ is constant for a block of message symbols, it is still random, resulting in random channel capacity. As a result, for fading channels, ergodic capacity, and outage probability are typically calculated.

The ergodic capacity is defined as the statistical average of the mutual information where the expectation is taken over $|h|^2$ and is given by:

$$C_{erg}=E\left\{\log_2\left(1+\frac{A_0^2M_k^2|h|^2(P_{k-1}+\sigma_v^2)}{\sigma_\zeta^2}\right)\right\}. \quad (31)$$

By Jensen's inequality applied to (31):

$$E\left\{\log_2\left(1+\frac{A_0^2M_k^2|h|^2(P_{k-1}+\sigma_v^2)}{\sigma_\zeta^2}\right)\right\}\le\log_2\left(1+\frac{A_0^2M_k^2|h|^2(P_{k-1}+\sigma_v^2)}{\sigma_\zeta^2}\right) \quad (32)$$

Where $E\{.\}$ stands for expectation.

### 3.1.2 Outage Capacity of AFCS

Capacity with an outage applies to slowly varying channels in which the instantaneous SNR is constant over a block of symbol transmissions and then changes based on the fading distribution. If the channel has an SNR $Q_h^2$ during a burst, then the capacity is given by $\log_2(1+Q_h^2)$, but as the transmitter is unaware of this SNR value, it must set a transmission rate that does not depend on the received SNR. In such a case, if the rate fixed by the transmitter is greater than $\log_2(1+Q_h^2)$, then the reliable transmission cannot be guaranteed. To take into consideration, capacity with an outage is considered. It allows the symbols to be transmitted with some probability of incorrect decoding. The transmitter, in particular, sets a minimum received SNR value and fixes the rate of transmission to $\log_2(1+Q_{hmin}^2)$. The symbols will be correctly decoded if received SNR $Q_h^2$ is greater than $Q_{hmin}^2$, otherwise outage will happen. The probability of an outage is given by:

$$p_{out} = p(Q_h^2 < Q_{hmin}^2), \quad (33)$$

And the capacity with outage consideration in bits/s/Hz is given as:

$$C_{out} = (1-p_{out})\log_2(1+Q_{hmin}^2), \quad (34)$$

Where $Q_{hmin}^2$ is a design parameter.

## 4. RESULTS AND DISCUSSION

The algorithm corresponding to AFCS with SISO Rayleigh fading channel model is implemented. Simulations are performed in MATLAB based on the results found in sections 2 and 3. A random input signal with mean $x_0 = 1$, variance $\sigma_0^2 = 0.625$ , and band limited to $2.5\, kHz$ is used for simulations, this signal is sampled at a frequency of 8 $kHz$ and each sample is held at the input of comparator for $n$ cycles. The complex Rayleigh fading coefficient is realized and it remains the same for $n$ cycles. Noise is considered AWGN in both forward and feedback channels with zero mean.

MSE achieved in each iteration is plotted in *figure 3*. The simulation is performed for $n = 20$. MSE for the proposed communication system architecture (AFCS) converges to zero even in presence of Rayleigh fading. The MSE per iteration versus the number of iterations for different values of AWGN variance in the forward channel shows that the MSE takes more iterations to converge to zero for higher noise variance. If the channel noise variance is low, MSE converges to zero in fewer iterations. As a result, the number of iterations required to reliably transmit the symbols is determined by the forward channel's SNR.

The algorithm corresponding to AFCS with SISO Rayleigh fading channel *figure 4* shows the effect of feedback noise variance on the MSE per iteration. The plot shows that for feedback noise variance $\sigma_v^2 = 10^{-2}$ and above there is not much change in the MSE per iteration performance but for $\sigma_v^2 = 10^{-1}$ the number of iterations required for MSE convergence to zero is higher. *Figure 5* shows the dependence of threshold number of cycles $(n^*)$ on forward channel SNR. This dependence is shown in *equation (29)* in *section 3.1* the plot in *figure 5* shows a decrease in $n^*$ as channel SNR improves.

The plot showing the realization of *equation (32)* for the AFCS case is shown in *figure 6*. It shows that the ergodic capacity for AFCS in block fading Rayleigh channel is less compared to the AFCS AWGN channel as expected. This difference can be made small or can be overcome by introducing diversity or MIMO techniques.

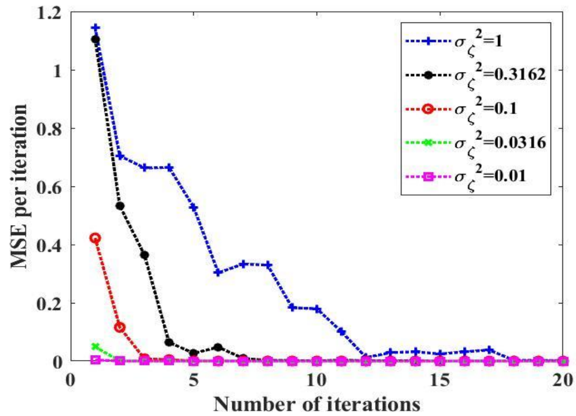


**Figure 3:** MSE per iteration achieved in number of iterations for different values of noise power in the feedback channel.

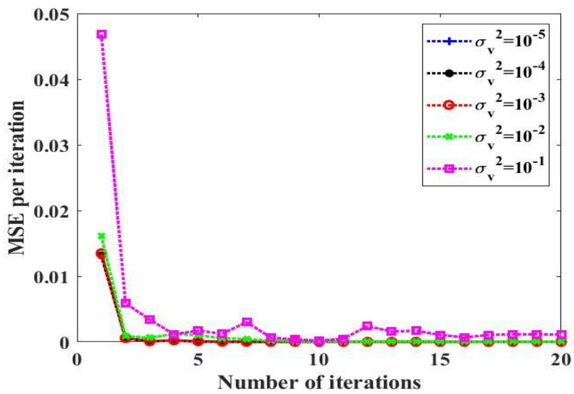


**Figure 4:** MSE per iteration achieved in number of iterations for different values of noise power in the feedback channel

In this paper, we are dealing with an analog communication system based on the Rayleigh fading channel model. As most of the message processing stages and performance objectives used in AFCS and digital communication system (DCS) are different, a one-to-one comparison of AFCS with DCS may not be fair. MSE is the performance criterion in AFCS, and the system as a whole is optimized to reach the least possible MSE, as shown in *section 3*. On the other hand, in DCS, there are many interrelated performance criteria, such as bit rate, power-bandwidth efficiency, bit error rate (BER), etc., and achieving an optimal balance of these objectives involves trade-offs. For example, increasing the bit rate increases the BER; to get a low BER, a higher power must be expended; and so on. Furthermore, DCS entails multiple stages such as digitization, source coding/decoding, channel coding/decoding, modulation, etc., and optimizing each of these stages for a common objective is challenging, resulting in a very complex, power-consuming system. Instead of complex coding, the AFCS employs feedback and iterative estimation algorithms. As a result, it will be a better choice for resource-constrained applications such as

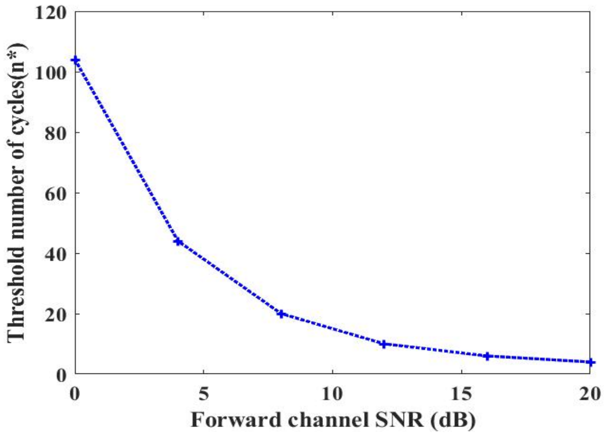


**Figure 5:** Dependence of threshold number of cycles on forward channel SNR

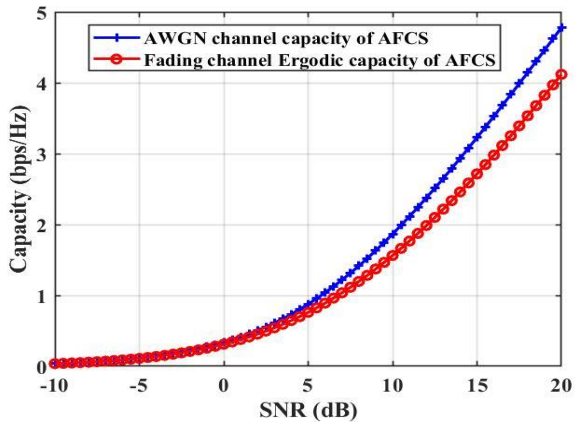


**Figure 6:** Ergodic capacity of AFCS compared with AWGN capacity of AFCS

Satellite communication, wireless sensor networks, and a multitude of other applications which rely on sensors powered by batteries. In applications where the source data is originally analog, for ease of processing and transmission DCS converts the analog signal to digital and then employs complex coding decoding algorithms to increase the data rate and combat noise. In this process of analog to digital conversion DCS loses information to quantization. Instead, AFCS suggests the transmission of un-coded analog information and efficiently uses the available communication resources with the help of feedback, which is available in most communication scenarios. There is undoubtedly an iterative procedure and feedback involved, which can be viewed as an overhead, but as illustrated in *figure 3*, the diversity scheme only requires 2-3 iterations to achieve close to zero mean square error, and most importantly the simplicity of the transmitter is very attractive in AFCS.

A low BER does not guarantee that an analog signal will be transmitted reliably in DCS as the quantization loss cannot be avoided. However, In AFCS transmission, quantization is not part of the process. Hence a very low or nearly zero MMSE means the exact reconstruction of the message at the receiver.

A specific channel SNR is used as a design parameter in the design of digital communication stages. DCS performance suffers greatly if the channel conditions are worse than that. However, if the channel conditions are better than the design parameter, the performance does not improve. This is known as the cliff effect [28], and it has an impact on DCS. In contrast, AM in AFCS is iteratively adjusted to channel conditions, so there is no cliff effect and there is an elegant deterioration in performance, in the case of bad channel quality.

## 5. CONCLUSION

This study has provided a comprehensive analysis of AFCS using a flat Rayleigh fading channel model. It has also provided insights related to the convergence of MSE to zero in the iterative algorithm with the help of a feedback channel. Furthermore, the dependence of the threshold number of cycles on the noise in the forward and feedback channels is discussed and simulated. The expressions for Rayleigh fading AFCS ergodic capacity and outage capacity are given, and a comparison with AWGN AFCS capacity is shown. Future work will involve the investigation of diversity and MIMO techniques for reducing the effect of fading and improving capacity.

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