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Design of a Multi-loop PI Controller for Minimum Phase System Level Regulation in a Quadruple Tank: A Method for Constraint Optimization

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ABSTRACT- A nonlinear optimization based decentralized PI controller for Two Input Two Output (TITO) is presented in this paper. Modelling of Quadruple tank minimum phase system with time delay is introduced here. The basic principles of nonlinear optimization are utilized to design the proposed PI controller in which the overshoot is bounded with constraints on the maximum closed-loop amplitude ratio, maximum closed loop width, gain and angle bounds. Besides, the control algorithm is designed for decoupled systems to reduce the loop interactions. Further, the first order plus dead time (FOPDT) model is derived for each of the decoupled subsystems to design the control law. The robust stability is analyzed by using Hurwitz and Upper triangular matrix of interrelated Kharitonov polynomials. The performance of the proposed control strategy is verified by MATLAB/SIMULINK software. The performance specifications of the simulation indicate that the suggested control strategy is more efficient.

Keywords: MIMO System, Decouplers, Model Reduction, Robustness.

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1. INTRODUCTION

Design of a controller and robust stability are two major challenges of MIMO system. We can employ centralized & decentralized methods for controlling MIMO system. The Centralized controllers suffer from a drawback of complex structure and lack of collaboration among the components of a system. Hence, the decentralized controllers are most widely used than centralized controllers because they offer simple structures and a smaller number of tuning parameters. Multivariable processes consist of a greater number of loops which are highly interacted. Decouplers are used to minimize the interaction between the loops. Over a decade different centralized controllers are designed based on several decoupling approaches and various techniques. We can employ either static decoupling or dynamic decoupling. Based on static decoupler design a centralized PI controller is devised which will use SSG matrix of the system because it is easy to tune controller parameters [1]. The static decouplers are simple to design but do not provide tolerable performance if there is a huge variation in transfer function elements. Hence different new decoupling methods are designed by obtaining an equivalent transfer function about ERGA [2], RNGA and RARTA [3], open loop process system function [4]. Inverted decoupling technique has many practical advantages such as simple to design but it would not be used in procedures that have RH side zeros because of poor stability. The crosswise elements of the decoupler are put to one in an enhanced form of simplified decoupling [5]. The decoupling methodology shows the limitations of getting stable decoupler elements for processes with time delays. Hence an innovative Internal Model Control based PID Commander is devised for Multi Output Multiple Input systems with numerous delays by splitting the process model into delay part and remaining part as filter [6]. Various methods are proposed in the literature to tune centralized PI controllers. The performances of tuning PI controller using Tanttu and Liestehto and Davison methods are compared [7]. Tanttu and Liestehto shows better performance in terms of time domain specifications. Relative Gain Array is frequently used to examine how the loops interact. The synthesis methodology [8] is used to create a concentrated PI controller for the TITO system, and it exhibits satisfactory performance when the RGA element is smaller than one. Sometimes it is very difficult to control unstable SISO processes along dead times because the performance specifications are larger. A novel centralized commander is devised for TITO Process along dead times based on ERGA-Transfer function matrix [9] using phase margin and gain margin specifications. The initial sequence process with time delay model parameters of the centralized PI Controller for the TITO system is estimated using a least squares optimization technique. [10]. When compared to PID Controllers, fractional order PID controllers are now employed the most in process industries since they offer time response performance. For the Conical Tank Process, the Harmony search method is used to create and fine-tune a centrally located fractional order PID controller [11].

Most MIMO processes have numerous time delays and higher order kinematics. The dominant pole concept in these time delay process is essential in deciding the steadiness of the supervisory system. Finding the multiloop controller's parameters for the TITO System to arrange the roots of the characteristic equation at corresponding locations is challenging. A novel method known as the "root trajectory method" has been devised to find the solution of characteristic polynomials[12].This method solve pole placement problem by identifying the sites where the root paths interact with real axis . Another strategy for creating a controller for these processes is to lower the order of the model to FOPDT. A direct synthesis-based approach is described for drafting a decentralized PID controller [13]. To obtain a standard PID controller mathematical series dependent strategy is accustomed. MIMO system dynamics are occasionally commanded using Single Input Single Output controllers. Because of the severe reciprocal action in the middle of loops, design of such controllers will encounter lot of difficulties. Hence configuration of multiloop controller is disintegrated into composing of controllers for several equivalent open loop processes. A model-based method is introduced for two loop systems [14]. This method is used for synthesis of PI controller and tuning parameters are articulated about utmost proportion of input, output, and occurrence of loops. The multiple loop governing framework can be broken down into several distinct loops which are expressed by correlated EOL system functions. This EOL system function is approximated to reduced form through coefficient matching based model reduction technique [15] and the controller is tuned by Internal model control-PID procedure. For controlling MIMO processes multiloop PID controller is generally selected because it is simple to design. The interplay between the loops causes this controller to perform poorly. To overcome this problem a multiscale control scheme is presented by decomposing the designated plant into few elementary modes where a separate controller is employed for each mode [16].

To tune PID controller various approaches have been developed over the decades. Among them analytical approaches have shown better performance. Here an Iterative Linear Matrix Inequality tuning approach based PID controller is designed for controlling Quadruple tank system [17]. A Sturdy segregated PID Controller is designed based on graphical tuning procedure to acquire stable regions of Proportional Integral Derivative commander specifications [18]. Some PID Controller tuning techniques are appropriate for certain types of processes, such as first- and second-order processes. A special kind of decentralized controller based on desired dynamic equation has been proposed to stabilize a category of Systems comprising of several inputs and outputs [19]. The usage of model predictive regulate with decentralized control strategy provides desired closed loop stability for multivariable process with less number of control loops. In order to achieve desired stability for quadruple tank process a decentralized multiparametric quadratic programming based MPC is presented[20].Design of a separate controller for each subsystem using decentralized control approach exhibits degraded performance when the system has nonminimum phase right half plane zeros. Closing of the loop through one module will present right half plane zero in other modules will pose a limitation on performance. Therefore, the necessary conditions for zero crossings are derived [21] to address above problem. These separate SISO controllers cannot completely squash the combined interactions of MIMO process. Hence combined sliding mode and reference conditioning technique [22] is presented to set boundaries for loop interactions. To overcome the difficulty of reducing interaction in a multivariable process a decentralized controller is presented based on equivalent system operate in relation to Efficacious Respective Acquire Array. When developing a controller, different gain and angle combinations of a certain loop as further loops have closed are considered [23]. The literature reports a variety of decoupling methods to lessen the connection between the loops. Among them optimal dynamic dividing approach has the advantages of working effectively at all frequencies. Based on this technique All loops use the same multivariable controller [24]is developed for TITO system. Although a PID controller can produce a convinced closed-loop response, as documented in the above-mentioned research articles, there is a bartering between robustness and performance. Various strategies have previously been put forth for creating a PID controller that is distributed. A strong PID controller that is distributed for the region is designed and stability regions of commander parameters which will assure the $H\infty$ sensitivity criteria are decided [25].

This work focuses on designing a blueprint for distributed control by considering optimization of disordered constraints. The most important performance standard is obtained by choosing parameters such as the closed-loop amplitude ratio and bandwidth. A larger number for Amplitude Ratio can result in a considerably quicker response. But the system’s efficiency will decrease. Additionally, a higher bandwidth number can greatly reduce the settling time. A careful selection of the values for AR and bandwidth is necessary to obtain the intended closed-loop performance. Furthermore, overshoot reduced by levy limitations on amplitude ratio while robustness is guaranteed by considering the constraints on phase and gain margins. The settling period is so reduced and a noticeably faster reaction is achieved by increasing the bandwidth. Verification of the controller's performance is conducted on industry benchmark TITO systems.

2. CONCEPTUALIZATION OF THE ISSUE

The conceptualization of the control problem is presented in this segment. The abstract interpretation for a Two Input Two Output structure is given by

$$o_1(s) = C_{1_1}(s)u_1(s) + C_{1_2}(s)u_2(s) \quad (1)$$
$$o_2(s) = C_{2_1}(s)u_1(s) + C_{2_2}(s)u_2(s)$$

Where $C_{11}(s) = c_{11}(s)e^{-\mu 11(s)}$ $C_{12}(s) = c_{12}(s)e^{-\mu 12(s)}$, $C_{21}(s) = c_{21}(s)e^{-\mu 21(s)}$ and $C_{22}(s) = c_{22}(s)e^{-\mu 22(s)}$ represents the transfer functions for the TITO system. Because changes in one input frequently impact the output of the other, loop interaction is a prevalent problem in TITO systems. The specifications of many variable structures can vary during the real-time operation. Due to disturbances and other parameter uncertainties, configuring a controller for a many variable structures are a challenging process. Hence, the primary objective is to design a control algorithm that ensures the loop performance under any situation.

2.1 Reduced Order POPLT Model

The strategies for model deduction are intended to be employed to get a Primary Order Process with Late Time model structure. The process dynamics are approximated using the Primary Order Process with Late Time prototype. This model makes it simple to calculate the gain, dead time, and time constant. The minimized model can roughly be described as

$$\xi_{ll}(s) = \frac{g_{ll}e^{-\sigma_{ll}s}}{T_{ll}s+1}, l=1,2 \quad (2)$$

Hence, at two points ω=0 and ω = ωjj frequency response fitting is gained to find out the unknowns, where ωjj is the angle crossover frequency

$$\xi_{jj}(0) = D_{jj}(0) \quad (3)$$

$$|\xi_{jj}(j\omega_{cjj})| = |D_{jj}(j\omega_{cjj})| \quad (4)$$

$$\angle\xi_{jj}(j\omega_{cjj}) = \angle Djj(j\omega_{cjj}) \quad (5)$$

Thus, the specifications of the Primary Order Process with Late Time are established as

$$g_{ll} = D_{jj}(0) \quad (6)$$

$$T_{jj} = \sqrt{\frac{g_{ll}{}^2 - |D_{jj}(j\omega_{cjj})|^2}{|D_{jj}(j\omega_{cjj})|^2 \omega_{cjj}{}^2}} \quad (7)$$

$$\sigma_{jj} = \frac{\pi + \tan^{-1}(-\omega_{cjj}T_{jj})}{\omega_{cjj}T_{jj}} \quad (8)$$

3. MODELLING OF QUADRUPLE TANK STRUCTURE WITH TIME DELAY

The Symbolic representation of Quadruple Tank Structure with time delays is exhibited in *figure 1*. The QTS has four tanks Tank1,Tank2,Tank3,Tank4 with four pumps Pump1, Pump2, Pump3, Pump4.The main goal of controlling QTSwDT is used to control the water levels h_1 in Tank1 and h2 in Tank2.The control input voltages v1 and v2 are bisected between pumps to give X_k(k=1,2,3,4),the input voltage to pump k and lagged at time t by dead times Δ_j(j=1,….,4) as mentioned below.

$$X_1 = \gamma_1 v_1(t-\Delta_1)$$
$$X_2 = \gamma_2 v_2(t-\Delta_2)$$
$$X_3 = (1-\gamma_2)v_2(t-\Delta_3)$$
$$X_4 = (1-\gamma_1)v_1(t-\Delta_4)$$

where $\gamma_1,\gamma_2 \epsilon (0,1)$

Let us consider that each pump k the flow rate given as $u_i x_i - u_{i2}$ where u_i, ui_2 are constants related with pump k. Based on weight equilibrium equations and Bernoulli's law the nonlinear prototype of QTSwDT expressed as

$$\begin{aligned}
\dot{h}_1 &= -\frac{b_1}{B_1}\sqrt{2gh_1} + \frac{b_3}{B_1}\sqrt{2g(h_3)} + \frac{(\gamma_1 u_1 v_1(t-\Delta_1) - u_{12})}{B_1} \\
\dot{h}_2 &= -\frac{b_2}{B_2}\sqrt{2gh_2} + \frac{b_4}{B_2}\sqrt{2g(h_4)} + \frac{(\gamma_2 u_2 v_2(t-\Delta_2) - u_{22})}{B_2} \\
\dot{h}_3 &= -\frac{b_3}{B_3}\sqrt{2gh_3} + \frac{((1-\gamma_2)u_3 v_2(t-\Delta_3) - u_{32})}{B_3} \\
\dot{h}_4 &= -\frac{b_4}{B_4}\sqrt{2gh_4} + \frac{((1-\gamma_1)u_4 v_1(t-\Delta_4) - u_{42})}{B_4}
\end{aligned} \quad (9)$$

This nonlinear model can be converted into linear model by state space equations. The transfer matrix from i too, R(s) provided as follows

o(s)=r(s)i(s)

Where $r(s) = \begin{bmatrix} R_{11}(s) & R_{12}(s) \\ R_{21}(s) & R_{22}(s) \end{bmatrix} =$

$$\begin{bmatrix} \frac{(\gamma_1 T_1 u_1)e^{-s\Delta_1}}{(B_1)(1+sT_1)} & \frac{(1-\gamma_2)T_1 u_3 e^{-s\Delta_3}}{(B_1)(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1)T_2 u_4 e^{-s\Delta_4}}{(B_2)(1+sT_4)(1+sT_2)} & \frac{(\gamma_2 T_2 u_2)e^{-s\Delta_2}}{(B_2)(1+sT_2)} \end{bmatrix} \quad (10)$$

The Quadruple Tank Process can be examined at two operating points *i.e.* minimal stage (MS) and non-minimal stage (NMS) characteristics. These characteristics are depending on the values of γ_1, γ_2. When $1>\gamma_2+\gamma_1>0$ the QTP is operated at non minimal stage and for minimal stage $\gamma_1+\gamma_2$ ϵ (1,2). The specifications of QTSwDT are displayed in *table 1*.

Table 1. QTSwDT SPECIFICATIONS

$B_j(Cm^2)$	$b_j(cm^2)$	$u_j(cm^3/(s.V))$	$u_{j2}(cm^3/s)$	j	Δ_j
20	0.2	3.75	3.5	4	9
12	0.2	3.90	3.07	3	7
20	0.26	3.83	2.0	2	6
12	0.26	3.80	2.71	1	5

For minimum stage (MS) operation of QTSwDT the values of γ_1,γ_2 are chosen as 0.65,0.60. By substituting the values of γ_1,γ_2 and parameter values of *table 1* in *equation-10* the transfer matrix r(s) is obtained as follows.

$$r(s) = \begin{bmatrix} \frac{1.585e^{-0.5s}}{(7.685*s+1)} & \frac{e^{-0.7s}}{(4.778*s+1)(7.685*s+1)} \\ \frac{0.778e^{-0.9s}}{(3.705*s+1)(11.85*s+1)} & \frac{1.362e^{-0.6s}}{(11.85*s+1)} \end{bmatrix} \quad (11)$$

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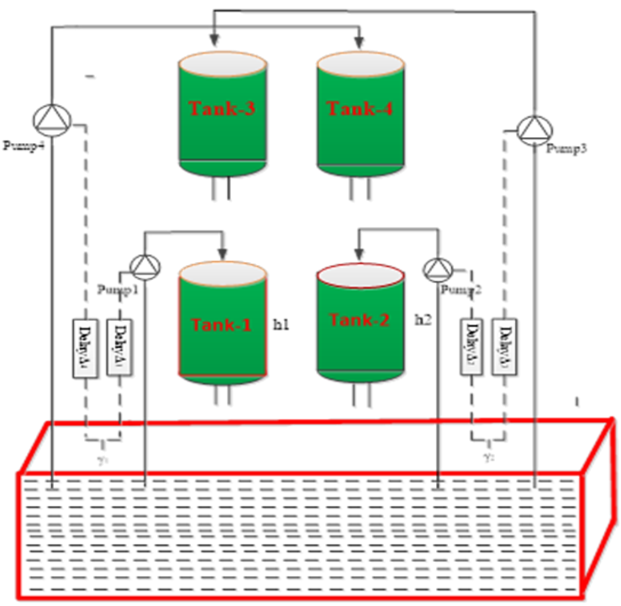

Figure 1. Quadruple Tank Process with time delays

4. MULTILOOP CONTROLLER DESIGN BASED ON NONLINEAR OPTIMIZATION

In this section, we delve into the design aspect of the PI controller. The decentralized PI controller is designed with the nonlinear constraints in mind. Additionally, by placing restrictions on highest shut-loop ratio amplitude HT, the overshoot is diminished.The Primary Order Process with Late Time prototype can be described by following equation.

$$\xi(s) = \frac{g_{cl}e^{-\sigma s}}{Ts+1} \tag{12}$$

The system function for the PID controller is provided by

$$F(s)=g_{pl}(1+\frac{1}{\tau_i s}) \tag{13}$$

Equations (12) and (13) are combined to obtain the unclose loop system function and is expressed as

$$W_0(s) = \frac{g_{cl}g_{pl}(\tau_i s+1)e^{-\sigma s}}{\tau_i s(\tau s+1)} \tag{14}$$

Applying frequency analysis to *equation (14)*, the fraction of amplitude Γ0 and the angle variation β0 are obtained as

$$\Gamma_{(s)} = \frac{g_{cl}g_{pl}\sqrt{1+(\omega\tau_i)^2}}{\omega\sqrt{(\omega^2\tau^2+1)\tau_i^2}} \tag{15}$$

$$\beta_0 = \begin{cases} -90^0 - \tan^{-1}(T\omega) - \omega\alpha + \text{ó}\ (\omega), & \text{if ó } (\omega)\text{greaterthan or equal to } 0 \\ 90^0 - \tan^{-1}(T\omega) - \omega\alpha + \text{ó}\ (\omega), & \text{if ó } (\omega)\text{lessthan } 0 \end{cases} \tag{16}$$

Where $\sqcap\ (\omega) = \tan^{-1}(\omega\tau_i)$

The closed cycle's system function is $W_{Cl} = \frac{W_0}{1+W_0}$

The fraction of amplitude assessed as

$$\text{ó}_{Cl} = \frac{1}{\sqrt{(\frac{1}{\text{ó}_0} + \tan\sigma_0)^2 + \sin^2(\sigma_0)}} \tag{17}$$

The following equation can be used to determine the frequency of phase crossover

$$\sqcap_{Cl}\ (\omega_c) = 0.707 \tag{18}$$

The closed loop amplitude ratio's highest value H_T can be represented as

$$H_T = \max(\sqcap_{Cl}\ (\omega)), \forall\omega \tag{19}$$

The gain bound and angle bound of unfastened loop system function W0 are represented as

$$A_m = \frac{1}{|W_0(j\omega_p)|} \tag{20}$$

$$\sigma_m = \pi + \angle W_0(j\omega_g) \tag{21}$$

where $|W_0(j\Omega_p)|$ = one, $\angle W_0(j\Omega_g) = -1800$ and Λ_p and Ω_g are phase, gain traverse frequencies correspondingly. Replace *equations (14)-(15)* in (19)-(20), then it is obtained as

$$A_m = \frac{\Lambda_p\tau_i}{g_{cl}g_{pl}}\sqrt{\frac{\tau^2\Lambda_p+1}{1+\Lambda_p\tau_i^2}} \tag{22}$$

$$\sigma_m = \begin{cases} \text{ó}\ (\Omega_g) - \theta\omega - \tan^{-1}(\Omega_g T) - 90^0, & \text{if ó } (\Omega_g) \geq 0 \\ \text{ó}\ (\Omega_g) - \theta\omega - \tan^{-1}(\Omega_g T) + 270^0, & \text{if ó } (\Omega_g) < 0 \end{cases} \tag{23}$$

$$\frac{g_{cl}g_{pl}}{\Omega_g\tau_i}\sqrt{\frac{1+(\Omega_g\tau_i)^2}{\Omega_p^2\tau^2+1}} = 1 \tag{24}$$

$$\sigma_p = \begin{cases} \text{ó}\ (\Lambda_p) - \tan^{-1}(\Lambda_p T) - \omega\theta - 90^0, & \text{if ó } (\Lambda_p) \geq 0 \\ \text{ó}\ (\Lambda_p) - \tan^{-1}(\Lambda_p T) - \omega\theta + 270^0, & \text{if ó } (\Lambda_p) < 0 \end{cases} \tag{25}$$

The *equations (22) to (25)* cannot be resolved directly due to unknown variables (Ωg), (Λp), (g_{Cl}), (Ti) and (Td). The optimization problem is therefore formulated by limiting the maximum closed loop bandwidth, maximum amplitude ratio, gain and angle bounds. The optimization issue is as follows.

$$\max_{\Omega_g,\ \Omega_p, g_{cl}, g_{pl}, \tau_i} \omega_c$$

$$\text{ó}_{Cl}\ (\omega_c) = 0.707$$

$$A_m \geq i_m^*$$

$$\sigma_m \geq s_m^*$$

$$H_T \geq U_m^*$$

The lower limits of amplitude bound and angle bound are represented by i_m^* and s_m^* respectively, while the upper bound maximum amplitude ratio is denoted by U_m^*. On the other hand,

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the bounds for gain margin and phase margin, A_m and σ_m, are related and can be expressed as follows.

$$A_m \geq 1+\frac{1}{H_T} \tag{26}$$

$$\sigma_m \geq 2\sin^{-1}\left(\frac{1}{2H_T}\right) \tag{27}$$

The obtained PI controller parameters are as follows

$$g_{\mathrm{pl}} = \frac{\omega_p T}{A_m g_{\mathrm{cl}}} \tag{28}$$

$$T_i = \frac{1}{\frac{1}{T}-\frac{4\omega_p\sigma}{\pi}+2\omega_p} \tag{29}$$

Where $$\omega_p = \frac{\frac{1}{2}\pi(A_m\text{-}1)+A_m\sigma_m}{(A_m^2\text{-}1)\omega} \tag{30}$$

For a given TITO System as in *equation-11*, the FOPDT models using *equation-12* are

calculated as follows

$$Q_{11}(s)=1.0942\frac{e^{-0.5075s}}{(1+5.321s)} \tag{31}$$

$$Q_{22}(s)=0.7908\frac{e^{-0.6109s}}{(1+6.909s)} \tag{32}$$

Solving *equations (26)-(27)* for A_m=3 & σ_m=400 the recommended nonlinear Optimization

Proportional Integral Commander realized as

$$C(s) = \begin{bmatrix} 2.089+\frac{1}{0.5910s} & 0 \\ 0 & 3.088+\frac{1}{0.7186s} \end{bmatrix} \tag{33}$$

5. ROBUST STABILITY ANALYSIS

Robust stability is the important criteria to determine stability of the control system when it is subjected to unknown perturbations. This stability can be analyzed by Kharitonov polynomials When the parameters of the system are not known accurately. These polynomials are used to judge the system's stability. Once the characteristic polynomial of the process is determined, the Hurwitz-Routh criterion is applied to decide stability of the system. The characteristic polynomial of Four Tank Process with Delay Time is presented by

$$\det[Kc(s)Kp(s) + I]=0 \tag{34}$$

For Four Tank Process with Delay Time, the *equation-26* can be expressed as

$$113.7375s^4+125.38s^3+34.5119s^2+1.5537s^1+0.0561=0 \tag{35}$$

The above equation is looks similar to equation of

$$l_0 s^0+l_1 s^1+l_2 s^2+l_3 s^3+l_4 s^4=0 \text{ with}$$
$$m_i \leq l_i \leq n_i \quad i=0,1,2……n \tag{36}$$

As outlined in [13], the four Kharitonov polynomials are as follows

$$\begin{aligned} U^1(s)&=m_0+m_1s+n_2s^2+n_3s^3+m_4s^4 \\ U^2(s)&=m_0+n_1s+n_2s^2+m_3s^3+m_4s^4 \\ U^3(s)&=n_0+m_1s+m_2s^2+n_3s^3+n_4s^4 \\ U^4(s)&=n_0+n_1s+m_2s^2+m_3s^3+n_4s^4 \end{aligned} \tag{37}$$

We can form the Hurwitz matrix(M) and Upper triangular matrix (Mn) for the first Kharitonov polynomial {U1(s)} is as shown below

$$M = \begin{bmatrix} m_1 & m_0 & 0 & 0 \\ m_3 & m_2 & m_1 & m_0 \\ 0 & m_4 & m_3 & m_2 \\ 0 & 0 & 0 & m_4 \end{bmatrix} M_n = \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ & m_{22} & m_{23} & m_{24} \\ & & m_{33} & m_{34} \\ & & & m_{44} \end{bmatrix}$$

For *equation (33)* the obtained Hurwitz matrix (m) and Upper Triangular matrix (Mn) is given in *table 2*.

Table 2. Hurwitz (m) and upper triangular (M_n) matrix of interrelated Kharitonov polynomials

S. No	Polynomial	M	M_n	Comments
1	$U^1(s)$	$\begin{bmatrix} 1.3473 & 1 & 0 & 0 \\ 0.01366 & 0.3708 & 1.3473 & 1 \\ 0 & 0.00049 & 0.01366 & 0.3708 \\ 0 & 0 & 0 & 0.00049 \end{bmatrix}$	$\begin{bmatrix} 1.3473 & 1 & 0 & 0 \\ 0 & 0.3606 & 1.3473 & 1 \\ 0 & 0 & 0.0118 & 0.36943 \\ 0 & 0 & 0 & 0.00049 \end{bmatrix}$	Hurwitz stable
2	$U^2(s)$	$\begin{bmatrix} 0.90193 & 1 & 0 & 0 \\ 0.01366 & 0.24826 & 0.90193 & 1 \\ 0 & 0.00049324 & 0.01366 & 0.24826 \\ 0 & 0 & 0 & 0.00049324 \end{bmatrix}$	$\begin{bmatrix} 0.90193 & 1 & 0 & 0 \\ 0 & 0.2331 & 0.90193 & 1 \\ 0 & 0 & 0.01175 & 0.2461 \\ 0 & 0 & 0 & 0.00049324 \end{bmatrix}$	Hurwitz stable

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3	$U^3(s)$	$\begin{bmatrix} 1.1023 & 1 & 0 & 0 \\ 0.01117 & 0.24826 & 1.1023 & 1 \\ 0 & 0.00049324 & 0.01117 & 0.24826 \\ 0 & 0 & 0 & 0.00049324 \end{bmatrix}$	$\begin{bmatrix} 1.1023 & 1 & 0 & 0 \\ 0 & 0.2381 & 1.1023 & 1 \\ 0 & 0 & 0.00888 & 0.246188 \\ 0 & 0 & 0 & 0.00049324 \end{bmatrix}$	Hurwitz stable
4	$U^4(s)$	$\begin{bmatrix} 1.1023 & 1 & 0 & 0 \\ 0.0166 & 0.24826 & 1.1023 & 1 \\ 0 & 0.00049324 & 0.0166 & 0.24826 \\ 0 & 0 & 0 & 0.00049324 \end{bmatrix}$	$\begin{bmatrix} 1.1023 & 1 & 0 & 0 \\ 0 & 0.233 & 1.1023 & 1 \\ 0 & 0 & 0.0142 & 0.2461 \\ 0 & 0 & 0 & 0.00049324 \end{bmatrix}$	Hurwitz stable

6. OUTCOMES AND CONVERSATION

In segment simulation results are embodied to assess how well the suggested controller performs by using MATLAB /SIMULINK environment. The results are categorized into setpoint tracing and disturbance rejection tracing. In setpoint tracing initially step input of h_1 and h_2 is specified at t=0 sec and the corresponding results are displayed in *figure 2(a)* and *figure 2(b)*. Similarly in disturbance rejection tracing a step change of di=0.5 is introduced and the interrelated results are displayed in *figure 3(a)* & *figure 3(b)*.

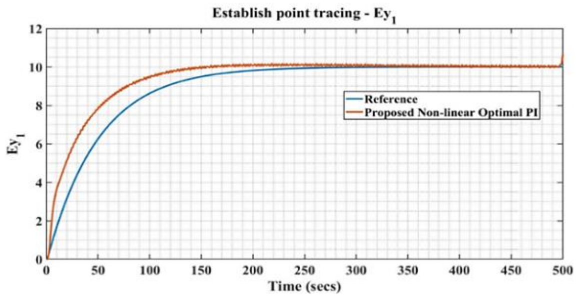

Figure 2(a). Establish point tracing -Change of liquid level in Tank1(Ey1)

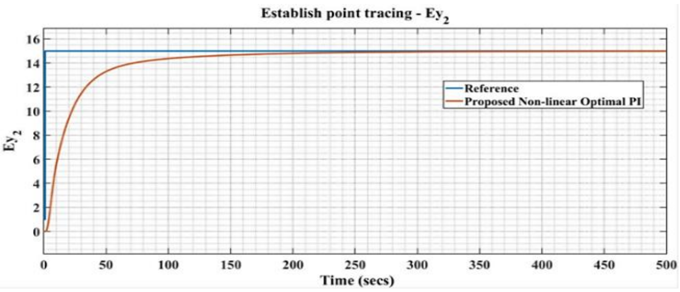

Figure 2(b). Establish point tracing -Change of liquid level in Tank2(Ey2

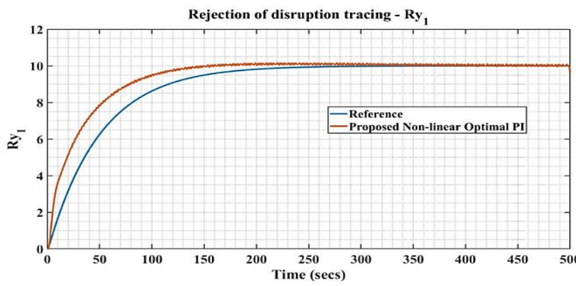

Figure 3(a). Rejection of disruption tracing -Change of liquid level in Tank1(Ry1)

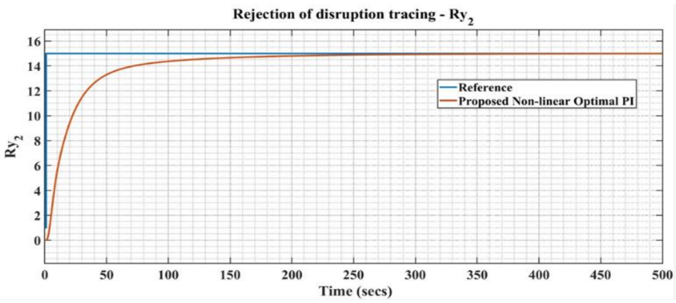

Figure 3(b). Rejection of disruption tracing -Change of liquid level in Tank2(Ry2)

From the below responses the settling times of proposed PI controller, Robust stability region based decentralized PI controller [25] for different input-output pairings with respect to Establish point tracing and Rejection of disruption tracing are provided in *table.3*.

Table 3. Performance Specifications

	Input(r)-Output(y)	Ts(Sec)-Proposed	Ts(Sec) MahapatroSR [25]
Establish point tracing	r1-y1	280	340
	r2-y2	300	350
Rejection of disruption tracing	r1-y1	320	360
	r2-y2	310	340

7. CONCLUSION

This paper proffered distributed PID controller that ensures scheme specifications through nonlinear optimization. The primary advantage of the suggested PI controller is flexibility, even though angle bound and gain bound provide the foundation for robustness. The robustness and set-point tracking of the closed-loop system are ensured by imposing boundary conditions on the amplitude ratio, leading to the attainment of the PI controller parameters. To showcase the effectiveness of the proposed controller, the TITO Quadruple Tank minimum phase framework was utilized for simulation purposes and the results were obtained. The resilience of the suggested system is demonstrated through the utilization of Kharitonov polynomials.

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