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International Journal of Electrical and Electronics Research (IJEER)
Research Article | Volume 12, Issue 3 | Pages 1109-1119 | e-ISSN: 2347-470X

State Estimation of Radial Distribution Systems Based on Multiple Legendre Neural Networks

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ABSTRACT- The conventional weighted least square (WLS) method is the most effective technique used in the state estimation of high voltage transmission system. Unfortunately, the application of WLS in radial distribution network encounter difficulties due to the inherent characteristics of these systems, such as the low measurement redundancy and high r/x ratio of the distribution systems. Given the structure of bulky systems that require a bulky number of measurements, the use of artificial neural networks is considered an effective alternative to estimate these values using a lesser number of measurements than conventional techniques. Due to state estimation based on ANN technique, the time-consuming gain matrix manipulation and pseudo measurements required in the conventional WLS method are no longer necessary. The efficiency of deep learning neural networks such as multi-layer perceptron (MLP) and Legendre neural network (LeNN) depends on the position of the measurements and the number of neural networks applied. Determining the applicable number of neural networks to ensure high estimation accuracy plays an important role in the estimation process. This aspect is addressed in this study, where multiple neural networks are used to improve performance compared to a single neural network. The results obtained indicate that determining the applicable number of neural networks depends on several factors such as the position of the measurements and the diversity of the data. The application of LeNN on state estimation of a 69-bus radial distribution network is used as an illustrative example to explain the distinctive feature of the proposed technique.

Keywords: State estimation, radial distribution system, multi-layer perceptron (MLP), Legendre neural network (LeNN), multiple neural networks.

ARTICLE INFORMATION
Author(s): Haider Hakim Sachit and Kassim Al-Anbarri;
Received: 11/07/2024; **Accepted**: 27/08/2024; **Published**: 30/09/2024;
e-ISSN: 2347-470X;
Paper Id: IJEER 1107-09;
Citation: 10.37391/ijeer.120347
Webpage-link:
https://ijeer.forexjournal.co.in/archive/volume-12/ijeer-120347.html

crossref member
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1. INTRODUCTION

Prediction of the distribution system states is vital for the real-time operation and control of modern distribution automation. State estimation is the basic tool for performing this task. Given a set of redundant and real but imperfect measurement, the power system state estimator gives the best estimate of the state variables (bus voltage magnitudes and angles) based on a selected statistical criterion. There are effective conventional methods like weighted least square WLS and weighted least absolute value estimator WLAV [1], [2]. These methods work well for estimating the states of transmission networks. There are limitations when applying these techniques in the state estimation of radial distribution systems [3]. The radial distribution networks have numerous nodes and laterals. It is impractical to collect data from all the nodes of the network, instead several meters are collecting the data at the substation (main source). Some meters are placed on critical point in the network. Most measurements are pseudo measurements and also virtual measurements. The main challenge these systems face is the difficulty in monitoring every part of the network due to its enormous size and many feeders. This renders parts of the distribution system unobservable. Several attempts have been carried out to improve the performance of the conventional WLS to efficiently solve the radial distribution system state estimation. An improvement to the state estimation of distribution system was suggested in Ref. [4] by using a current-based formulation. The burden of computation is reduced in a current-based state estimation because of the constancy of gain matrix which is derived from the bus admittance matrix. Lin and Teng [5] presented a decoupled version of the current-based state estimation. Some researchers solved the state estimation problem in radial distribution system by using a branch-based WLS state estimation [6-8]. The estimation is performed on a single branch of the network at a time. The estimation of states of the complete network is obtained by using a forward/backward sweep scheme. Haughton and Heydt [9] introduced a linear state estimator for smart distribution systems based on the linearization of the measurement function. In Ref [10] a state estimation algorithm was proposed to best estimate the states of a power distribution feeder based on the power summation load flow. The feeder was reduced to a series of a small equivalent two node sections and the estimation process stat sequentially from the source section to the load. Jose et al. [11] proposed an improvement to the distribution system state estimation by using synthetic measurements. An algorithm for a state estimation in active distribution network based on using

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symmetrical component domain was introduced in Ref. [12]. Naka et al. [13] addressed this problem as an optimization problem by using artificial intelligence techniques (hybrid particle swarm). The objective function to be minimized is the square of the errors between the measured and true quantities.

In all the forementioned literature, the accuracy of the estimation process is mainly dependent on the weights assigned to the pseudo measurements. These pseudo measurement values are generated from historical data or from normalized daily load profile. The inaccuracy of the generated pseudo measurement samples deteriorates the performance of the estimation process. Additionally, the forementioned approaches do not completely solve other issues arise when using conventional WLS method like the ill-conditioning of the Gain Matrix [14]. In this context, state estimation based on ANN can alleviate these limitations. Artificial neural networks have been used in well-known literature [15] to model spurious measurements. In Ref. [16], ANN was employed to clean up bad data to increase the accuracy of system state estimation. A different study [17] estimated the state of the system by utilizing machine learning to estimate unobserved components. Furthermore, some studies have estimated states using original components, as reported in [18]. In a comparable manner, a recent study [19] determined the measurements accessible to the system by using fully connected feed-forward artificial neural networks. As expressed in [20], neural networks have been used in studies to forecast the system state as opposed to estimating it. Some studies have demonstrated that neural networks can be thought of as fully connected linear equations [21-23]. In Ref [21], the neural network optimization technique was employed by removing the weights that were least responsive to back propagation based on using the neural network pruning concept. Pao and Philips [24] introduced a different type of neural network called functional link artificial neural network (FLANN) to overcome real-life challenges in determining the number of hidden layers and the number of nodes for each layer. FLANN offers several advantages, including a simpler structure and lower computational complexity due to having fewer parameters compared to traditional neural network models. Mall and Chakraverty [25] explored the application of a new structure of ANN called Legendre neural network (LeNN) which is based on FLANN. LeNN does not have a hidden layer. It is worthy to note that all the previous literature estimates the magnitude of the bus voltage, the voltage angle is not considered.

The contribution of this paper is the building of a machine learning architecture that effectively tackles the problem of state estimation in radial distribution systems. The proposed technique is based on using multiple Legendre neural networks. To the best of our knowledge, this technique has not yet been documented in the literature. The estimation process is achieved accurately with a low number of measurements and without using pseudo or virtual measurements.

In this paper, the following aspects of using artificial neural networks are explored:

1. Evaluation of the performance of the Legendre Neural Network (LeNN) in comparison with multi-layer perceptron (MLPs) and traditional techniques such as WLS in estimating a radial distribution system's state (bus voltage magnitude and angle).
2. Assessment of the effect of using multiple neural networks in place of a single neural network.
3. discussing the impact on the accuracy of the results of splitting the various neural networks needed for voltage and angle calculation.

The remaining paper is structured as follows. *Section 2* presents a quick review of the basic principle. Implementation of the ANN model is described in *section 3*. *Section 4* explores the results obtained by applying the ANN model to the 69-bus radial distribution system. Conclusion remarks are given in *section 5*.

2. REVIEW OF THE BASIC PRINCIPLE

In this paper, a set of three meters are placed in selected locations of the distribution networks to collect the real measurements of active and reactive power and the current. To training the ANN, the network is divided into several parts, each ANN is trained separately. The rules of placing the meters and the selection of the ANN are adopted as per Ref. [26]. A review of the basic principle is revisited in this paper. Figure 1 shows a simple radial distribution network where Z_a and Z_b represent the impedance of the main supply part, and I_a and I_b are the current in each busbar. In this simple system, measurements are used at point (C) of both real and imaginary power flow and the current passing through the feeder has a triple measurement.

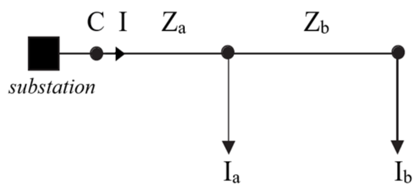

Figure 1. A simple radial distribution network

To illustrate the use of a triple measurement (P, Q, I), let us assume $I_{load} = I_a$, $I_b = 0$, the real (P) and imaginary (Q) power at the load point (a) by adding the complex power losses at Za, then $I = I_{load}$ .On the other hand, if $I_b = I_{load}$ and $I_a = 0$, also the real (P) and imaginary (Q) power at the load point (b) by adding the complex power losses at Z_a and Z_b, then $I = I_{load}$. But *if* $I_{load} = I_a+I_b$, the real (P) and imaginary (Q) power at the load point (a, b) by adding the complex power losses in Z_a and Z_b, then the current $I = I_{load}$.

Based on the effective use of triple measurements (P, Q, I) as a tool to understand and improve the performance of radial distribution networks, we can choose a suitable structure and location of measurement. The following principles [26] are adopted in determining the structure and locations of meters.

- It is preferable to take triple measurements at the beginning of the main feeder, where that these points contain the total network load, which shows the feeder’s performance accurately and comprehensively.

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- the locations for triple measurements can also be identified at the beginning of each major lateral, as these measurements contribute to monitoring the load at the level of the laterals and identifying any potential problem in these areas.
- if there are short, close laterals, it is better to take triple measurements from the feeder equipped for those laterals instead of each small lateral, to reduce the number of measurements used.

Using these rules, it is possible to determine the optimal location for the triple measurement (P, Q, I) in radial distribution networks, providing comprehensive and realistic data that helps in analysing and evaluating network performance and reducing the number of measurements and maintenance costs.

3. IMPLEMENTATION OF THE NEURAL NETWORKS MODEL

3.1 multi-layer perceptron (MLP)

Multi-layer perceptions (MLPs) have been used in previous studies to improve their performance in predicting the states of radial distributed systems. A MLP containing one hidden layer with a fixed number of neurons was used, which is characterized by full connectivity between all input, hidden, and output layers as shown in *figure 2*, referred to in Reference [2], [15-23].

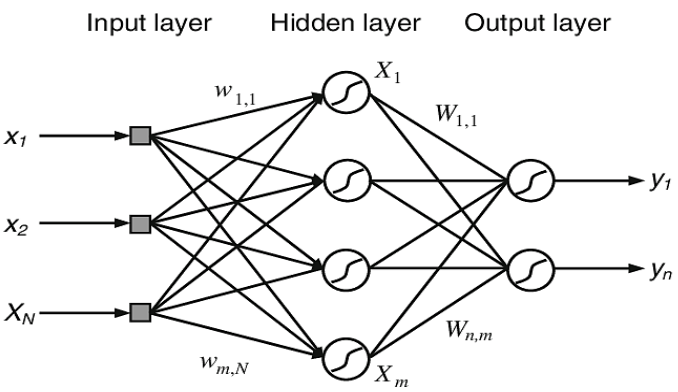

Figure 2. Multi-layer perceptions (MLPs)

An optimization algorithm represented by the "Adaptive Moment Algorithm" (Adam) [27] was used in the process of training the multi-layer perceptions (MLPs), using TensorFlow, NumPy and PyTorch etc. frameworks to build the multi-layer perceptron (MLP). The Python programming language was used to implement this process. The effective functions of the hidden layer and output layer are given respectively in the indicated equations (1) and (2).

$$y = \mathrm{sigmoid}(x) \tag{1}$$

$$y = x \tag{2}$$

In the above equations, $(\boldsymbol{x})$ and $(\boldsymbol{y})$ represent the input and output respectively, where a sigmoid function was used for the hidden layer and a linear function for the output layer.

3.2 Legendre neural network (LeNN)

Legendre neural networks (LeNNs) consist of only an input layer and an output layer, without a hidden layer in between, which reduces the need to specify the number of neurons. In LeNN the hidden layer is replaced by a functional expansion block for enhancement of the input patterns using Legendre poly-nominals [25] that each input datum is expanded to several terms using Legendre polynomial. Where a Legendre polynomial based functional link Artificial neural network (FLANN). The zero and first order Legendre polynomial are represented in Equations (3) and (4) respectively.

$$L_0(x) = 1 \tag{3}$$

$$L_1(x) = x \tag{4}$$

The recursive formula to generate higher order Legendre polynomials is represented in *equation (5)*

$$L_{n+1}(x) = \frac{1}{n+1}[(2n+1)xL_n(x) - nL_{n-1}] \tag{5}$$

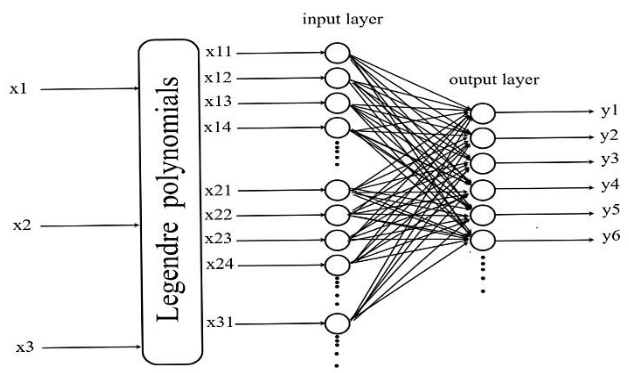

Figure 3. Legendre neural networks (LeNNs)

Where n is the order of Legendre polynomials. An optimization algorithm represented by the "Adaptive Moment Algorithm" (Adam) [27] was used in the process of training the Legendre neural network (LeNN), also using TensorFlow, NumPy and PyTorch etc. frameworks to build the Legendre neural network (LeNN). Also, The Python programming language was used to implement this process. The effective function of the output layer is given in the *equation (6).*

$$y_i = tanh(x_i) \tag{6}$$

To measure the discrepancy between the actual and predicted values from the neural networks (NNs), mean square error (MSE), which represents a measure of error and reflects the general error in predictions, is represented in *equation (7).*

$$MSE = \frac{1}{RE}\sum_{r=1}^{R}\sum_{e=1}^{E}(y_e^r - \hat{y}_e^r)^2 \tag{7}$$

On the other hand, used the accuracy as a percentage for measure the performance the ANNs model, that tolerance level is 0.3%, which is represented by *equation (8).*

$$ACC = \frac{1}{RE}\sum_{r=1}^{R}\sum_{e=1}^{E}1(|y_e^r - \hat{y}_e^r| \leq \epsilon) \times 100 \tag{8}$$

Where R represents the number of training or testing samples, E the number of features for each sample, y_e^r and $\hat{y}_e^r$ the truth and predictor values respectively, e represents the target feature, r the number of features for each the target sample, ϵ is the tolerance level, $1(|y_e^r - \hat{y}_e^r| \leq \epsilon)$ is an indicator function that equals 1 if the absolute difference between the predicted and true value is within the tolerance level ϵ, and 0 otherwise.

4. SIMULATION RESULTS AND DISCUSSION

In this part of the study, we examine the efficiency and effectiveness of applying the proposed technique to a Legendre neural network (LeNN) and compared with multi-layer perceptron (MLP) and WLS technique. This technique was applied to 69-busbar radial distribution system, where a single neural network was used in the first case and a group of small neural networks in the second case. After that, the results were compared between them after training and testing.

4.1. Test 69-bus radial distribution system

To confirm the efficiency and effectiveness of the proposed methodology, we used a large system consisting of a 69-busbar radial distribution network, and the data for this system were obtained from references [28].

The schematic diagram of the system appears in Figure 4, which includes a main feeder and seven laterals. Details of the system are as follows: 1) the main feeder emanating from the substation, which extends from node 1 to 27; 2) lateral no. 1, which emanates from node 3 and extends to node 35; 3) lateral no. 2, which emanates from node 3 and extends to node 46; and 4) lateral no. 3, which emanates from node 4 and extends to node 50; 5) lateral no. 4, which emanates from node 8 and extends to node 52; 6) lateral no. 5, which emanates from node 9 and extends to node 65; 7) lateral no. 6, which emanates from node 11 and extends to node 67; 8) lateral no. 7, which emanates from node 12 and extends to node 69. In this system, and based on the rules mentioned in the second section, triple measurements (P, Q, I) were taken at the beginning of each main or lateral feeder to describe in detail: a) triple measurements at bus 1 between node 1 and 2, b) triple measurements at bus 10 between node 10 and 11, and In order to improve the efficiency of the data collection process and reduce the number of triple measurements required, triple measurements for small lateral that are close together (laterals no. 6 and 7) are collected by placing triple measurements for the nearby bus (at bus 10).This procedure is done to reduce the total number of triplicate measurements. , c) triple measurements at bus 3 between node 3 and 28, d) triple measurements at bus 3 between node 3 and 36, e) triple measurements at bus 9 between node 9 and 53, since the number of triple measurements (P, Q, I) is 5 ,the sum is (3 x 5 = 15), it is assumed that there are 15 measurements available in the system, While using the WLS method, *table 1* selects the location of actual, virtual, and pseudo measurements by take profile of the loads for previous years.

Table 1. Location and error of each measurement

Type measurement	position (bus)	total	error	variance
Actual measurement (P,Q,I)	a,b,c,d,e	$3 \times 5 = 15$	10 %	1e-8
virtual measurement (P,Q)	2,5,15,19,23,25,30,31,32, 38,42,44,47,56,57,58,60,63	$2 \times 18 = 36$	no error	1e-12
pseudo measurements (P,Q)	6,7,8,9,10,11,12,13,16,17,18,20,21, 22,24,26,27,28,29,33,34,35,36,37, 39,40,41,43,45,46,48,49,50,51,52,53, 54,55,59,61,62,64,65,66,67,68,69	$2 \times 47 = 94$	20 %	1e-2

To demonstrate the efficiency of the proposed methodology, generate 50,000 samples by changing the load by a percentage ranging between 20-200% of the base load, with purpose generating random numbers between 0.2-2 and multiplying them by the base load and generating these samples using the load flow technique. There are two pair in the dataset: 10,000 samples for training and 40,000 samples for testing. Each sample has 69 outputs, which in this system are estimates of the voltage magnitude or angle, and 15 inputs, which are represented by the three measurements (P, Q, I). The long feeders in the system (the main feeder and its laterals) are divided into several artificial neural networks to increase the efficiency of training and testing, in addition to reducing the time spent training and testing these neural networks.

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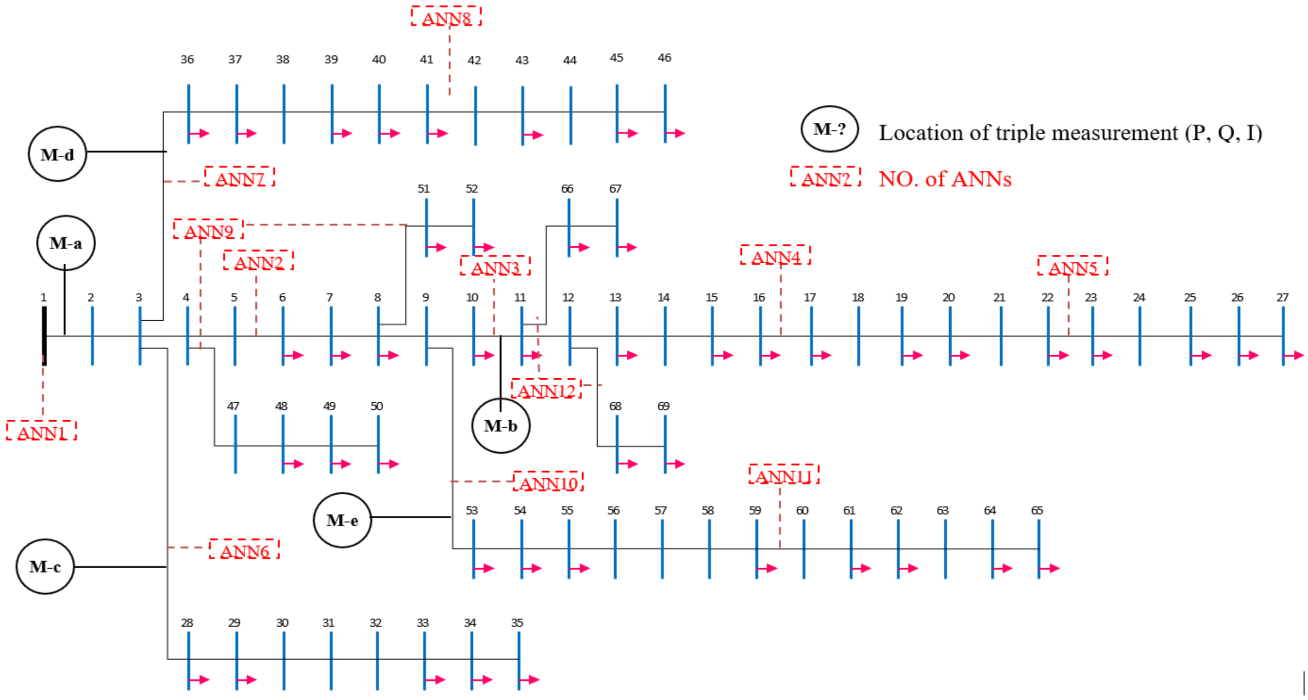

Figure 4. 69-bus radial distribution system

Twelve neural networks were used in this system: five neural networks for the main feeder and seven neural networks for the laterals, as follows: ANN1 covers from node 1 to 5 and uses triple measurements of type (a); ANN2 covers from node 6 to 10 and uses triple measurements of type (a); ANN3 covers from node 11 to 16 and uses triple measurements of type (b); ANN4 covers from node 17 to 22 and uses triple measurements of type (b); ANN5 covers from node 23 to 27 and uses triple measurements of type (b); ANN6 covers from node 28 to 35 and uses triple measurements of type (c); ANN7 covers from node 36 to 41 and uses triple measurements of type (d); ANN8 covers from node 42 to 46 and uses triple measurements of type (d); ANN9 covers from node 47 to 52 and uses triple measurements of type (a); ANN10 covers from node 53 to 59 and uses triple measurements of type (e); ANN11 covers from node 60 to 65 and uses triple measurements of type (e); ANN12 covers from node 66 to 69 and uses triple measurements of type (b). Each neural network was trained for 500 epochs, with the aim of increasing the efficiency and accuracy of estimation and improving the overall system performance. The multi-layer perceptron (MLP) shown in *figure 2* has a single hidden layer with five neurons that are applied to all other neural networks in the system. The output layer's effective function employs a linear function, whereas the hidden layer's effective function makes use of a sigmoid function. As for the Legendre neural network (LeNN) shown in *figure 3*, it only includes an input layer and an output layer, without a hidden layer. The effective function of the output layer uses a hyperbolic tangent function, and Legendre polynomials [25] with several five were used to expand the input, so that the number of inputs becomes five instead of each input. To create a Legendre neural network (LeNN). The remaining information is contained in *table 2*.

Table 2. List of MLP and LeNN hyperparameters

Hyperparameters	MLP	LeNN
	Values	
Optimizer	Adam	
Batch size	32	
Number of epochs	500	
learning rate	5×10^{-4}	
type loss	MSE	
Activation function of output layer	Linear	Tanh
Activation function of hidden layer	Sigmoid	
Number of hidden layers	1	
hidden size	5	
Number of Legendre polynomials		5

Table 3. Comparative of ANNs performance in handling free error and noise for voltage magnitude

No. of ANN	Inputs (Meas.)	Outputs (Busbar)	Training / Testing			
			Legendre neural network (LeNN)		Multi-layer perceptron (MLP)	
			MSE	ACC (%)	MSE	ACC (%)
ANN1	(a)	1 to 5	2.88e-10 / 2.91e-10	100 / 100	3.45e-8 / 3.47e-8	100 / 100
ANN2	(a)	6 to 10	7.02e-7 / 6.93e-7	99.962 / 99.962	9.60e-7 / 9.55e-7	99.764 / 99.772
ANN3	(b)	11 to 16	7.30e-7 / 7.54e-7	99.70 / 99.677	1.38e-6 / 1.38e-6	99.197 / 99.194
ANN4	(b)	17 to 22	1.51e-6 / 1.54e-6	98.727 / 98.655	2.65e-6 / 2.69e-6	94.519 / 94.382
ANN5	(b)	23 to 27	1.73e-6 / 1.75e-6	97.954 / 98.007	2.48e-6 / 2.52e-6	95.024 / 94.864
ANN6	(c)	28 to 35	4.51e-9 / 4.47e-9	100 / 100	1.62e-8 / 1.61e-8	100 / 100
ANN7	(d)	36 to 41	1.17e-9 / 1.22e-9	100 / 100	2.58e-8 / 2.61e-8	100 / 100
ANN8	(d)	42 to 46	1.16e-8 / 1.18e-8	100 / 100	4.62e-7 / 4.63e-7	100 / 100
ANN9	(a)	47 to 52	4.49e-7 / 4.46e-7	100 / 100	7.56e-7 / 7.67e-7	99.947 / 99.931
ANN10	(e)	53 to 59	2.20e-7 / 2.20e-7	100 / 100	5.31e-7 / 537e-7	99.999 / 100
ANN11	(e)	60 to 65	3.99e-7 / 3.97e-7	99.997 / 99.995	8.85e-7 / 8.95e-7	99.940 / 99.938
ANN12	(b)	66 to 69	4.27e-7 / 4.45e-7	99.86 / 99.859	6.53e-7 / 6.42e-7	99.910 / 99.921
Average			5.15e-7 / 5.22e-7	99.683 / 99.681	9.03e-7 / 9.11e-7	99.025 / 99.00
Single ANN	(a,b,c,d,e)	1 to 69	1.01e-6 / 1.12e-6	97.995 / 97.721	1.10e-6 / 1.18e-6	97.502 / 97.575
WLS		1 to 69	MSE: 1.90e-05		ACC (%): 80.608	

Table 4. Comparative of ANNs performance in handling free error and noise for phase angle

No. of ANN	**Inputs (Meas.)**	**Outputs (Busbar)**	**Training / Testing**			
			Legendre neural network (LeNN)		**Multi-layer perceptron (MLP)**	
			MSE	**ACC (%)**	**MSE**	**ACC (%)**
ANN1	(a)	1 to 5	2.87e-9 / 2.92e-9	100 / 100	5.42e-9 / 5.47e-9	100 / 100
ANN2	(a)	6 to 10	4.78e-7 / 4.87e-7	99.994 / 99.998	5.30e-7 / 5.42e-7	99.982 / 99.994
ANN3	(b)	11 to 16	8.29e-7 / 8.53e-7	99.897 / 99.89	3.94e-6 / 3.94e-6	86.589 / 86.323
ANN4	(b)	17 to 22	1.22e-6 / 1.25e-6	99.442 / 99.411	4.34e-6 / 4.34e-6	84.365 / 84.298
ANN5	(b)	23 to 27	1.31e-6 / 1.34e-6	99.212 / 99.186	4.45e-6 / 4.45e-6	83.822 / 83.799
ANN6	(c)	28 to 35	7.23e-9 / 7.20e-9	100 / 100	9.08e-9 / 9.16e-9	100 / 100
ANN7	(d)	36 to 41	6.00e-9 / 6.21e-9	100 / 100	6.68e-9 / 9.67e-9	100 / 100
ANN8	(d)	42 to 46	2.51e-8 / 2.51e-8	100 / 100	350e-8 / 3.47e-8	100 / 100
ANN9	(a)	47 to 52	4.91e-7 / 4.88e-7	99.995 / 99.997	5.16e-7 / 5.09e-7	99.99 / 99.989
ANN10	(e)	53 to 59	2.04e-7 / 2.07e-7	100 / 100	2.89e-7 / 2.91e-7	100 / 100
ANN11	(e)	60 to 65	3.45e-7 / 3.47e-7	99.998 / 100	3.90e-7 / 3.94e-7	99.998 / 99.995
ANN12	(b)	66 to 69	6.75e-7 / 6.98e-7	100 / 99.998	3.61e-6 / 3.63e-6	88.294 / 88.035
Average			**4.66e-7 / 4.76e-7**	**99.878 / 99.873**	**1.51e-6 / 1.51e-6**	**95.253 / 95.203**
Single ANN	**(a,b,c,d,e)**	**1 to 69**	**1.01e-6 / 1.07e-6**	**97.997 / 96.886**	**1.11e-6 / 1.11e-6**	**97.712 / 97.721**
WLS		**1 to 69**	**MSE: 4.151e-06**		**ACC (%): 85.406**	

The results in *tables 3* and *table 4* show that relying on multiple neural networks leads to more accuracy in performance compared to relying on a single neural network in the system when using MLP and LeNN methods, also results of LeNN apparent more efficient and accuracy compared with MLP and WLS methods to estimate voltage magnitude and phase angle. It is noted that traditional methods often require a larger number of measurements, for example in this system 137 measurements are required, while 15 measurements are sufficient in the current approach.

4.2. Effect of Measurement Error on Neural Network

In previous cases, artificial neural networks were used for training and testing under ideal conditions, but real-world applications involve measurement noise and uncertainty. Each measurement has an intrinsic error margin and is dependent on the manufacturer's specifications as well as communication channels that transfer data from the measurement locations to the central computing station. The noise that results from data transfer through communication channels affects the precision of recorded values, underscoring the importance of comprehending how error and noise affect neural network efficacy. Within this context, artificial neural networks (ANNs) are subjected to evaluation across two distinct scenarios: firstly, wherein input data derived from measurements via communication channels for all test models is tainted by error and noise, and secondly, wherein input data for both training and test models encompasses error and noise. Presently, the prevailing maximum error threshold stands at less than ±2%. Nonetheless, this study endeavors to replicate the influence of error and noise within the data obtained from measurements by adopting a worst-case scenario approach, imposing a maximum measurement error of ±10% across all measurements. The multi-layer perceptron (MLP) and the Legendre neural network (LeNN) [25] underwent testing within the 69-buses radial distribution system only. Initially, these neural networks underwent training utilizing free-error and free-noise data, encompassing both single neural networks and multiple neural networks. Then, these networks' performance was tested with datasets that included measurement errors and noises.

The results of these analyses are given in *tables 5* and *table 6*. It is apparent that both single and multiple neural networks and the WLS method exhibited marked declines in performance and accuracy for voltage magnitude and phase angle when exposed to data containing errors and noise. Furthermore, all concerning details of the location of the measurements for WLS and neural networks, including the number of hidden layers, neurons per layer, training and testing sample sizes, polynomial order for LeNN, number of epochs, and activation functions per layer, remain the same as those delineated in the preceding section.

In the second case, neural network training was performed using error-tainted and noisy input data. The testing process was carried out in the presence of this error and noise, and the results are represented in *table 7* and *table 8* for voltage magnitude and phase angle. From the results, it is noted that training neural networks using input data contaminated with error and noise leads to a significant improvement in the efficiency and accuracy of all types of neural networks, whether single neural networks or multiple neural networks, compared to training them without the presence of this error and noise in the data. The results obtained indicate that neural networks (ANNs) should be trained in the presence of noise and error in the input data to minimize errors in the prediction of voltage magnitude. This is regarded as a practically acceptable performance for real-world applications compared with the WLS method, which gives practically unacceptable results in addition to many of the challenges mentioned previously when applied in the real world. Based on these results, it can be concluded that training neural networks (ANNs) in the presence of error and noise in the input data is necessary to reduce errors in predicting voltage magnitude and is considered a practically acceptable performance for practical applications.

The question that arises is the possibility of improving the performance of neural networks in terms of changing the number of hidden layers, the number of neurons for each layer, the number of models in training and testing, the polynomial number of the LeNN, the number of epochs, and the effective function for each layer. To answer this question, test the ANN11 by changing the number of hidden layer neurons for the MLP and the polynomial number for the LeNN, as in the *table 9*.

From *table 9*, it is seen that the performance of the multi-layer perceptron (MLP) improves slightly as the number of neurons in the hidden layer decreased to 2 compared to 5. Likewise, a slight improvement in the performance of the Legendre neural network (LeNN) is observed when the number of polynomials is decreased to 2 per input compared to 5, that mean the optimize the training and testing phases with reduced time requirements and computational complexity. Hence, optimizing both neural network methods by setting the number of neurons and expanders to two for enhances their performance in this application.

Table 5. Comparative of ANNs performance in handling error and noise tested for voltage magnitude.

No. of ANN	Inputs (Meas.)	Outputs (Busbar)	Training / Testing			
			Legendre neural network (LeNN)		Multi-layer perceptron (MLP)	
			MSE	ACC (%)	MSE	ACC (%)
ANN1	(a)	1 to 5	2.88e-10 / 1.88e-9	100 / 100	3.45e-8 / 1.51e-6	100 / 97.267
ANN2	(a)	6 to 10	7.02e-7 / 2.17e-6	99.962 / 95.124	9.6e-7 / 2.39e-5	99.764 / 60.092
ANN3	(b)	11 to 16	7.30e-7 / 3.17e-2	99.7 / 3.492	1.38e-6 / 3.68e-3	99.197 / 3.611
ANN4	(b)	17 to 22	1.51e-6 / 3.17e-2	98.727 / 3.352	2.65e-6 / 4.07e-3	94.519 / 3.399
ANN5	(b)	23 to 27	1.73e-6 / 2.23e-2	97.954 / 3.346	2.48e-6 / 4.60e-3	95.024 / 3.205
ANN6	(c)	28 to 35	4.51e-9 / 7.00e-9	100 / 100	1.61e-8 / 7.10e-8	100 / 100
ANN7	(d)	36 to 41	1.17e-9 / 3.54e-9	100 / 100	2.58e-8 / 4.21e-6	100 / 87.184
ANN8	(d)	42 to 46	1.16e-8 / 1.79e-8	100 / 100	4.62e-7 / 5.00e-7	100 / 100
ANN9	(a)	47 to 52	4.49e-7 / 1.09e-6	100 / 98.837	7.56e-7 / 1.44e-5	99.947 / 75.94
ANN10	(e)	53 to 59	2.20e-7 / 3.41e-4	100 / 18.591	5.31e-7 / 4.67e-6	99.999 / 83.932
ANN11	(e)	60 to 65	4.00e-7 / 5.86e-4	99.997 / 12.344	8.85e-7 / 1.75e-5	99.94 / 47.691
ANN12	(b)	66 to 69	4.27e-7 / 4.15e-2	99.86 / 3.541	6.53e-7 / 3.82e-3	99.91 / 3.589
Average			5.175e-7 / 1.00e-2	99.683 / 53.219	9.03e-7 / 1.35e-3	99.025 / 55.493
Single ANN	(a,b,c,d,e)	1 to 69	1.51e-6 / 3.45e-5	97.435 / 64.459	1.68e-6 / 5.45e-5	96.502 / 50.667
WLS		1 to 69	MSE: 5.05e-05		ACC (%): 65.812	

Table 6. Comparative of ANNs performance in handling error and noise tested for phase angle

No. of ANN	Inputs (Meas.)	Outputs (Busbar)	Training / Testing			
			Legendre neural network (LeNN)		Multi-layer perceptron (MLP)	
			MSE	ACC (%)	MSE	ACC (%)
ANN1	(a)	1 to 5	2.87e-9 / 5.01e-5	100 / 63.483	5.42e-9 / 1.91e-6	100 / 96.590
ANN2	(a)	6 to 10	4.78e-7 / 4.11e-4	99.994 / 27.377	5.30e-7 / 2.18e-6	99.982 / 95.734
ANN3	(b)	11 to 16	8.29e-7 / 1.43e-4	99.899 / 21.974	3.94e-6 / 9.42e-6	86.588 / 67.170
ANN4	(b)	17 to 22	1.22e-6 / 1.56e-4	99.441 / 20.909	4.34e-6 / 1.48e-5	84.365 / 62.566
ANN5	(b)	23 to 27	1.31e-6 / 1.26e-4	99.212 / 26.408	4.45e-6 / 1.19e-5	83.822 / 62.189
ANN6	(c)	28 to 35	7.23e-9 / 6.00e-5	100 / 51.188	9.08e-9 / 2.99e-8	100 / 100
ANN7	(d)	36 to 41	6.00e-9 / 1.96e-4	100 / 31.020	6.68e-9 / 4.22e-7	100 / 99.999
ANN8	(d)	42 to 46	2.51e-8 / 1.27e-4	100 / 22.698	3.5e-8 / 4.52e-6	100 / 85.673
ANN9	(a)	47 to 52	4.91e-7 / 9.31e-5	99.995 / 34.034	5.16e-7 / 8.45e-6	99.990 / 84.201
ANN10	(e)	53 to 59	2.04e-7 / 4.55e-5	100 / 38.003	2.89e-7 / 6.86e-6	100 / 76.405
ANN11	(e)	60 to 65	3.45e-7 / 5.23e-5	99.998 / 42.034	3.90e-7 / 1.20e-5	99.998 / 58.331
ANN12	(b)	66 to 69	6.75e-7 / 1.59e-4	100 / 21.753	3.61e-6 / 1.63e-5	88.293 / 58.661
Average			**4.66e-7 / 1.35e-4**	**99.878 / 33.407**	**1.51e-6 / 9.89e-6**	**95.253 / 78.959**
Single ANN	**(a,b,c,d,e)**	**1 to 69**	**1.26e-6 / 1.66e-4**	**97.993 / 21.696**	**1.60e-6 / 1.9e-5**	**96.24 / 56.442**
WLS		**1 to 69**	**MSE: 8.33e-06**		**ACC (%): 73.402**	

Table 7. Comparative of ANNs performance in handling error and noise trained and tested for voltage magnitude

No. of ANN	Inputs (Meas.)	Outputs (Busbar)	Training / Testing			
			Legendre neural network (LeNN)		Multi-layer perceptron (MLP)	
			MSE	ACC (%)	MSE	ACC (%)
ANN1	(a)	1 to 5	5.67e-10 / 5.76e-10	100 / 100	9.47e-9 / 9.2e-9	100 / 100
ANN2	(a)	6 to 10	1.38e-6 / 1.38e-6	98.512 / 98.537	2.5e-6 / 2.52e-6	93.649 / 93.461
ANN3	(b)	11 to 16	8.23e-6 / 8.21e-6	66.462 / 66.60	9.16e-6 / 9.03e-6	64.066 / 64.405
ANN4	(b)	17 to 22	9.39e-6 / 9.35e-6	63.861 / 63.942	1.13e-5 / 1.15e-5	60.464 / 60.597
ANN5	(b)	23 to 27	9.63e-6 / 9.58e-6	63.238 / 63.476	1.18e-5 / 1.20e-5	59.762 / 59.691
ANN6	(c)	28 to 35	4.73e-9 / 4.67e-9	100 / 100	4.92e-8 / 4.88e-8	100 / 100
ANN7	(d)	36 to 41	1.48e-9 / 1.54e-9	100 / 100	3.52e-7 / 3.53e-7	100 / 100
ANN8	(d)	42 to 46	1.45e-8 / 1.5e-8	100 / 100	1.1e-7 / 1.1e-7	100 / 100
ANN9	(a)	47 to 52	7.17e-7 / 7.17e-7	99.542 / 99.53	1.75e-6 / 1.77e-6	97.683 / 97.61
ANN10	(e)	53 to 59	3.11e-6 / 3.13e-6	90.456 / 90.242	4.23e-6 / 4.23e-6	86.85 / 87.061
ANN11	(e)	60 to 65	9.9e-6 / 1.0e-5	64.948 / 64.307	1.29e-5 / 1.32e-5	58.868 / 58.481
ANN12	(b)	66 to 69	7.64e-6 / 7.63e-6	68.089 / 68.199	7.84e-6 / 5.21e-6	67.404 / 67.894
Average			4.17e-6 / 4.17e-6	84.592 / 84.569	5.18e-6 / 5.21e-6	82.396 / 82.433
Single ANN	(a,b,c,d,e)	1 to 69	1.02e-5 / 1.04e-5	71.96 / 71.981	3.33e-5 / 3.35e-5	53.832 / 53.852
WLS		1 to 69	MSE: 5.05e-05		ACC (%): 65.812	

Table 8. Comparative of ANNs performance in handling error and noise trained and tested for phase angle.

No. of ANN	Inputs (Meas.)	Outputs (Busbar)	Training / Testing			
			Legendre neural network (LeNN)		Multi-layer perceptron (MLP)	
			MSE	ACC (%)	MSE	ACC (%)
ANN1	(a)	1 to 5	6.99e-9 / 7.00e-9	100 /100	2.39e-8 / 2.38e-8	100 / 100
ANN2	(a)	6 to 10	1.09e-6 / 1.11e-6	99.310 / 99.354	1.30e-6 / 1.32e-6	98.840 / 98.826
ANN3	(b)	11 to 16	4.25e-6 / 4.27e-6	84.710 / 84.551	5.21e-6 / 5.16e-6	80.233 / 80.425

ANN4	(b)	17 to 22	4.88e-6 / 4.89e-6	81.563 / 81.647	5.44e-6 / 5.51e-6	79.238 / 79.087
ANN5	(b)	23 to 27	5.03e-6 / 5.02e-6	80.890 / 81.006	5.38e-6 / 5.35e-6	79.552 / 79.695
ANN6	(c)	28 to 35	7.00e-9 / 6.97e-9	100 /100	3.02e-8 / 3.05e-8	100 / 100
ANN7	(d)	36 to 41	5.24e-9 / 5.30e-9	100 / 100	3.18e-8 / 3.18e-8	100 / 100
ANN8	(d)	42 to 46	4.69e-8 / 4.73e-8	100 / 100	7.39e-8 / 7.42e-8	100 / 100
ANN9	(a)	47 to 52	7.45e-7 / 7.47e-7	99.778 / 99.798	9.88e-7 / 9.95e-7	99.493 / 99.502
ANN10	(e)	53 to 59	2.87e-6 / 2.86e-6	91.092 / 91.070	3.15e-6 / 3.15e-6	89.929 / 89.990
ANN11	(e)	60 to 65	9.55e-6 / 9.53e-6	64.705 / 64.383	9.9e-6 / 9.85e-6	63.885 / 63.881
ANN12	(b)	66 to 69	3.96e-6 / 3.98e-6	86.106 / 86.027	4.48e-6 / 4.45e-6	83.613 / 83.713
Average			2.28e-6 / 2.71e-6	90.680 / 90.570	3.25e-6 / 3.00e-6	89.595 / 89.593
Single ANN	(a,b,c,d,e)	1 to 69	4.45e-6 / 4.42e-6	89.777 / 89.792	5.92e-6 / 5.95e-6	84.129 / 84.118
WLS		1 to 69	MSE: 8.33e-06		ACC (%): 73.402	

Table 9. performance of ANN11 for MLP and LeNN for different hidden nodes and Legendre polynomials.

No. of Legendre polynomials or hidden nodes	Training / Testing			
	Legendre neural network (LeNN)		Multi-layer perceptron (MLP)	
	MSE	ACC (%)	MSE	ACC (%)
2	9.80e-6 / 9.88e-6	65.35 / 64.727	1.05e-5 / 1.07e-5	63.675 / 65.445
4	9.89e-6 / 9.96e-6	64.953 / 64.491	1.33e-5 / 1.35e-5	58.738 / 58.351
5	9.90e-6 / 1.00e-5	64.948 / 64.307	1.29e-5 / 1.32e-5	58.868 / 58.481
6	9.94e-6 / 1.00e-5	64.888 / 64.214	1.11e-5 / 1.10e-5	61.239 / 61.488
8	9.94e-6 / 1.00e-5	64.995 / 64.250	1.123e-5 / 1.14e-5	61.945 / 61.499
10	8.73e-5 / 9.1e-5	51.398 / 51.008	1.21e-5 / 1.21e-5	58.661 / 59.041

4.3 Practical considerations

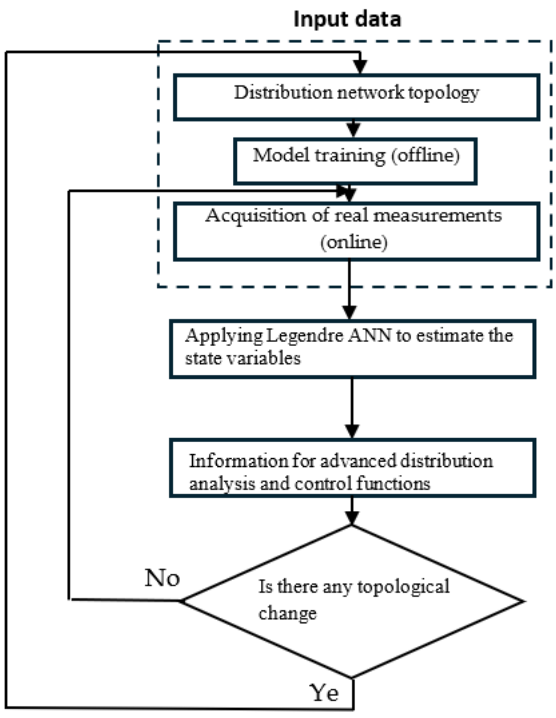

Figure 5. Practical structure of the proposed state estimation technique

Figure 5 demonstrates the practical implementation of the proposed state estimator in a radial distribution network. After collecting the system configuration data (node and branch connections, status of disconnector switches, meter's location, etc.), the ANN model is trained and tested (this task is carried out offline). The real-time measurements (only the telemetered measurements) at the main distribution substation and some critical locations in the network are acquired through dedicated communication channels. These data are fed to the ANN model to predict the state variables of the system. The output information from the estimator will form the basis for advanced network analysis and online control functions. Although the proposed state estimation scheme gives accurate results, it requires a pronounced amount of data to train the ANN model. This data can be generated by adopting a reliable load flow algorithm.

5. CONCLUSIONS

In this study, the efficiency of using the proposed technique to estimate voltage magnitude and angle in radial distribution systems is demonstrated. The results in the article indicate:

- The research suggests that using multiple artificial neural networks can enhance the system's efficiency and accuracy, compared to relying solely on a single neural network.
- The propose a LeNN model apparent performance and accuracy higher compared with MLP and WLS.
- The study shows that the results achieved are reasonable and acceptable even in the event of an error in the measurement data, which gives an advantage to the stability and reliability of the proposed method compared to traditional methods such as WLS in radial distribution system.

Supplementary Materials: There are no supplementary materials.

Author Contributions:
1. Haider Hakim Sachit - Conceptualization, methodology, software, validation, writing—original draft preparation
2. Kassim Al-Anbarri - Review and editing, visualization, supervision. All authors have read and agreed to the published version of the manuscript.

Funding Source: This research received no external funding.
Acknowledgments: The authors would like to thank the College of Engineering, Mustansiriyah University (www.uomustansiriyah.edu.iq) Baghdad-Iraq for providing the research facilities.

Conflicts of Interest: The authors declare no conflict of interest.

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