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Design of Sliding Mode Controller Combined with Nonlinear Disturbance Observer for Trajectory Tracking of Mobile Robots in Mixed Terrain

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ABSTRACT- Accurate trajectory tracking is a fundamental yet challenging requirement for mobile robots, especially when operating on surfaces with varying frictional characteristics (mixed terrain) and subjected to external disturbances as well as model uncertainties. This study presents the design and evaluation of an integrated control strategy aimed at enhancing trajectory tracking performance under these demanding conditions. A Sliding Mode Controller (SMC), known for robustness but prone to chattering, was integrated with a Nonlinear Disturbance Observer (NDO). The NDO was designed to estimate lumped disturbances encompassing varying friction, model errors, and other external disturbances; this estimate was then used for compensation within the SMC control law to reduce the switching component's amplitude. The effectiveness of the proposed SMC-NDO method was verified through simulations on a mobile robot model following a reference trajectory in a simulated mixed terrain environment under various disturbance conditions. Simulation results showed that the proposed SMC-NDO controller significantly improves trajectory tracking accuracy and reduces chattering compared to the traditional SMC controller and a PID controller. The integration of the NDO with SMC proves to be an effective approach for improving mobile robot trajectory tracking, enhancing accuracy and robustness while mitigating chattering in challenging environments with varying terrain and disturbances.

Keywords: Mobile robot, Nonlinear Disturbance Observer, Sliding Mode Control; Robust Control, Trajectory Tracking.

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1. INTRODUCTION

Mobile Robots (MRs) have become an indispensable part of many modern applications, ranging from automated logistics in warehouses [1], precision agriculture [2], to the exploration of hazardous environments and personal service systems. One of the most fundamental and crucial requirements for their effective operation is the ability to accurately follow a predetermined reference path – the trajectory tracking problem [3], a topic that has been extensively reviewed and studied [4]. This accuracy directly impacts mission success, operational safety, and the overall efficiency of the robotic system.

However, achieving high trajectory tracking performance for mobile robots faces significant challenges. Factors such as nonlinear dynamics and nonholonomic constraints inherent in many types of mobile robots [5], along with the presence of uncertainties in the dynamic model and disturbances from the external environment [6], complicate the control problem.

This challenge becomes even more severe when robots operate in mixed or uneven terrain environments [7], where sudden and unpredictable changes in the friction coefficient between the wheels and the ground, wheel slippage, and non-uniform surfaces can cause significant deviations from the desired trajectory [7, 8]. Ensuring robust trajectory tracking on such complex terrains is an active area of research, applicable to both wheeled and tracked robots [9]. Various control methods have been proposed to address the trajectory tracking problem. Classical controllers like PID often struggle with significant nonlinearities and disturbances. Advanced model-based techniques like Backstepping [10] require model information. Intelligent control methods such as fuzzy logic [11], neural networks [12], or hybrid approaches like adaptive fuzzy sliding mode control [13] can handle uncertainties but may involve complex designs. Model Predictive Control (MPC) is also an important research direction but demands high computational capabilities [14].

Among robust control methods, Sliding Mode Control (SMC), based on variable structure control theory [15, 16], stands out due to its ability to maintain stability and good performance in the presence of bounded uncertainties and disturbances. Numerous applications of SMC for mobile robots have demonstrated its effectiveness [17, 18], including variations like

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adaptive SMC and fractional-order SMC [19] to address specific issues such as actuator faults [19]. However, an inherent drawback of SMC is the chattering phenomenon – high-frequency oscillations in the control signal caused by the switching component [20], which not only reduces accuracy and wastes energy but can also cause wear on actuators [21]. Solutions like using boundary layers or higher-order SMC [22] can reduce chattering but often compromise robustness or increase complexity.

A promising approach is to combine SMC with disturbance observers (DO) [23] or similar estimation techniques like extended state observers (ESO) [24] or fault observers. The observer estimates the "lumped disturbance," including both external disturbances and internal uncertainties, in real-time. This information is used for compensation in the SMC control law, allowing for a reduction in the amplitude of the switching control component, thereby mitigating chattering while preserving robustness [25]. The SMC-DO combination has been successfully applied in various fields, but its application to mobile robots on mixed terrain, where friction and slippage are major and continuously varying sources of disturbance, requires further investigation.

This paper proposes an innovative integrated control strategy: Sliding Mode Controller combined with a Nonlinear Disturbance Observer (SMC-NDO) for the trajectory tracking problem of mobile robots operating in mixed terrain environments.

A core highlight is the use of the NDO to effectively estimate the "lumped disturbance" in real-time. This disturbance includes all uncertain factors such as model errors, external disturbances, and particularly the complex, continuously changing effects of friction and wheel slip. By using the estimated disturbance value for active compensation within the SMC control law, the proposed controller significantly reduces the magnitude of the switching control component. This leads to the effective minimization of the chattering phenomenon—a major drawback of traditional SMC.

The main novelty of this work is the direct use of the estimated disturbance value ( ) for active compensation within the SMC control law. By doing so, the proposed controller can significantly reduce the magnitude of the switching control component (robustifying term), leading to the effective minimization of the chattering phenomenon – a major drawback of traditional SMC– helping to protect actuators and improve control smoothness. More importantly, accurate disturbance compensation helps to simultaneously maintain and enhance both the robustness and trajectory tracking accuracy of the system, even when facing the strong and rapidly changing disturbances characteristic of mixed terrain.

The main contribution of this paper is the presentation of the design, analysis, and verification (through detailed simulation) of a robust and smooth SMC-NDO control strategy, optimized for the challenging problem of mobile robot trajectory tracking on mixed terrain. Simulation results clearly demonstrate the superior performance of the proposed method compared to both traditional SMC and classic PID controllers in terms of both tracking accuracy and chattering suppression. Other advanced robust control methods are also being explored for similar problems.

The structure of the paper is organized as follows: *Section 2* presents the detailed mathematical model of the mobile robot and formulates the trajectory tracking control problem. *Section 3* describes the design of the disturbance observer and the integrated SMC-NDO controller, along with a preliminary stability analysis. *Section 4* presents the simulation results. Finally, *section 5* provides the conclusion.

2. SYSTEM MODELING

To design an effective trajectory tracking controller, it is first necessary to establish a mathematical model describing the kinematics and dynamics of the mobile robot. In this study, we focus on a differential drive mobile robot (DDMR) type with a structure as shown in *figure 1*.

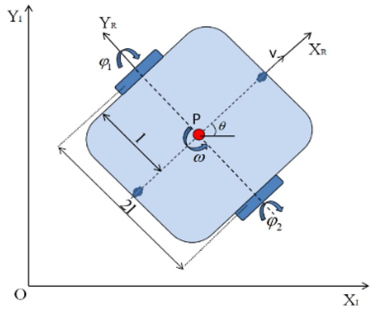

Figure 1. Structure of the mobile robot

2.1. Kinematic Model

The kinematic model describes the relationship between the robot's velocity in the global coordinate system and the control velocities, ignoring forces and inertia.

Coordinate Systems$\{OX_IX_I\}$**:** Global Frame (World Frame) $\{OX_IX_I\}$, a fixed coordinate system $\{O_RX_RX_R\}$which attached to the robot, origin O usually placed at the midpoint of the axis connecting the two driving wheels, X_R axis pointing forward.

Pose: The position and orientation of the robot in the global frame $\{OX_IX_I\}$ are defined by the state vector $q = [x, y, \theta]^T$, where *(x, y)* are the coordinates of the origin O_R, and θ is the orientation angle of the X_R axis relative to the X_I axis.

Control Velocities: DDMRs are typically controlled by linear velocity v (along the X_R axis) and angular velocity ω (around the Z-axis perpendicular to the plane of motion). The control velocity vector is $u_k = [v, \omega]^T$.

Kinematic Equation: The relationship between the time derivative of the state $\dot{q} = [\dot{x}, \dot{y}, \dot{\theta}]^T$ and the control velocities $u_k$is given by *equation (1)*:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (1)$$

2.2. Dynamic Model

The dynamic model considers inertia, friction forces, and other forces/torques acting on the robot, describing the relationship between control forces/torques and the robot's acceleration. Let be the robot's velocity vector. The dynamic model of a DDMR can be written in a general matrix form as *equation (2)*.

$$M\dot{\eta} + C(\dot{\theta})\eta + F(\eta) + \tau_d = B.u \qquad (2)$$

Where: M - Inertia matrix, symmetric, positive definite ($M = M^T > 0$); $C(\dot{\theta})\eta$ -Vector containing Coriolis and centrifugal terms;$F(\eta)$ -Vector describing known friction forces/torques;τ_d-Vector containing unmodeled dynamics, parametric uncertainties, and external disturbances. Includes: Complex friction (Coulomb, static) not modeled; u- Input control signal vector; B - Input matrix, relating the control signal u to the dynamics of η.

3. PROPOSED CONTROL STRATEGY

To effectively solve the trajectory tracking problem for mobile robots in mixed terrain, we propose a control strategy combining a Nonlinear Disturbance Observer (NDO) and a Sliding Mode Controller (SMC). The NDO is used to estimate the lumped disturbance d(t) acting on the system (including the effects of changing terrain), and this information is integrated into the SMC control law to enhance accuracy and minimize chattering. The structure diagram of the system is presented in *figure 2*.

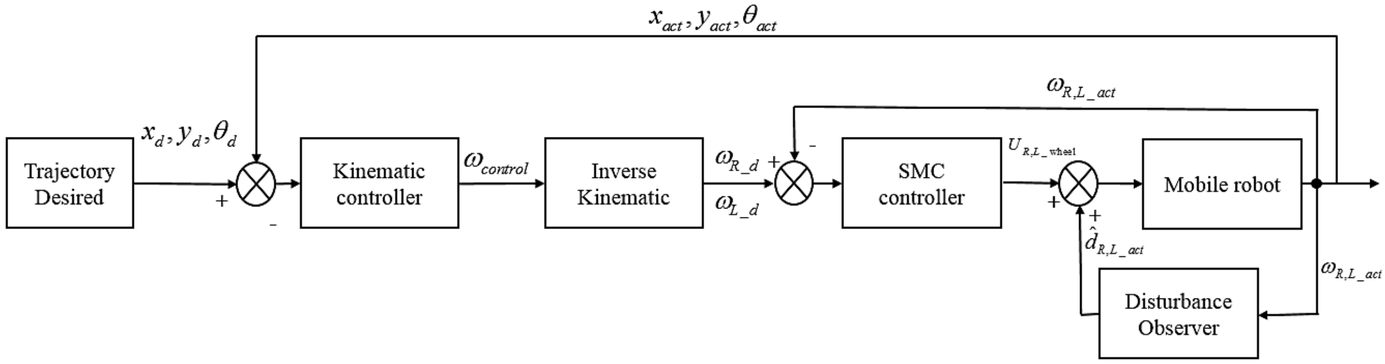

Figure 2. Structure diagram of the trajectory tracking control system for a mobile robot with SMC combined with a nonlinear disturbance observer

3.1. Nonlinear Disturbance Observer Design

The objective of the DO is to estimate the lumped disturbance vector in the dynamic model from *equation (2)*;

$$M\dot{\eta} + C(\dot{\theta})\eta + F(\eta) + \tau_d = B.u$$

We use a common Nonlinear Disturbance Observer (NDO) structure.

Definitions: Estimated disturbance vector:$\hat{d}$; Internal state vector of the observer:z; Observer gain matrix: $L(\eta)$, usually chosen as a positive definite diagonal matrix, where , with $\eta = [v \quad \omega]^T$ we have $L = \begin{bmatrix} l_v v \\ l_\omega \omega \end{bmatrix}$ with $l_v v, l_\omega \omega > 0$; Auxiliary vector: $p(\eta) = L(\eta)\eta$.

NDO Update Law: The estimated disturbance value is calculated by *equation (3).*

$$\hat{d}(t) = z(t) + L.\eta(t) \qquad (3)$$

The dynamics of the internal state are designed as *equation (4);*

$$\dot{z}(t) = -L.z(t) - L\big[p(\eta) + f(\eta, \dot{\theta}) + g.u\big] \qquad (4)$$

Equation (4) is implementable as it only requires measurable signals ($\eta, \dot{\theta}$often calculable from ω), known model components (f, g), the control signal u, and the internal state$z$.

Disturbance Estimation Error: Define the estimation error as calculated by expression *equation (5);*

$$e_d(t) = d(t) - \hat{d}(t) \qquad (5)$$

Analysis of the dynamics of this error shows:

$$\dot{e}_d(t) = -Le_d(t) - \dot{d}(t) \qquad (6)$$

From *equation (5)*, we see that if the disturbance $d(t)$ is constant or slowly varying ($\dot{d}(t) \approx 0$), then the error will converge to 0 at a rate determined by the gain matrix$L$. If changes (when the robot traverses different terrain regions), and if its derivative $\dot{d}(t)$ is bounded, then the estimation error $e_d(t)$ will be bounded and can be made small by choosing sufficiently large values for $l_v v$,$l_\omega \omega$. The choice L of needs to balance convergence speed and susceptibility to measurement noise amplification.

3.2. NDO-Based Sliding Mode Controller Design

The SMC is designed to control the velocity $\eta = [v \quad \omega]^T$ to track the reference velocity $\eta_d = [v_d \quad \omega_d]^T$.
Define the velocity error:

$$e_\eta(t) = \eta(t) - \eta_d(t) = \begin{bmatrix} v - v_d \\ \omega - \omega_d \end{bmatrix} \qquad (7)$$

Choose the Sliding Surface:

$$s(t) = e_\eta(t) = \eta(t) - \eta_d(t) \qquad (8)$$

Equation (8) aims to design the control law $u$such that $s(t)$ converges to 0, which implies $\eta(t)$ converges to$\eta_d(t)$.

SMC-NDO control law: To drive *s* to *0*, consider the derivative of s:$\dot{s}(t) = \dot{e}_\eta = \dot{\eta} - \dot{\eta}_d$

Substituting $\dot{\eta}$ from the dynamic *equation (2)* yields *equation (9)*.

$$\dot{s} = f(\eta, \dot{\theta}) + g(u) + d(t) - \dot{\eta}_d \quad (9)$$

The SMC-DO control law is designed to impose a desired dynamic on *s*. Setting the right-hand side of *eq. (9)* equal to the desired dynamics and replacing the actual disturbance *d(t)* with its estimate $\hat{d}(t)$ from the NDO *equation (3)* gives *equation (10)*.

$$f(\eta, \dot{\theta}) + g(u) + \hat{d}(t) - \dot{\eta}_d \approx -Ks\text{-}Q\,sgn(s) \quad (10)$$

where$K = diag(k_v, k_\omega)$ and $Q = diag(q_v, q_\omega)$ are positive definite diagonal gain matrices, and $sgn(s) = [sgn(s_v) \quad sgn(s_\omega)]^T$is the vector sign function.

Solving the above equation for the control signal *u* (assuming *q* is invertible, which is usually the case) yields *equation (11)*.

$$g(u) = -f(\eta, \dot{\theta}) - \hat{d}(t) + \dot{\eta}_d - Ks\text{-}Q\,sgn(s) \quad (11)$$

Thus, the SMC-NDO control law is:

$$u = g^{-1}[-f(\eta, \dot{\theta}) - \hat{d}(t) + \dot{\eta}_d - Ks\text{-}Q\,sgn(s)] \quad (12)$$

Analysis and Chattering Reduction: The control law in *equation (12)* includes:

Model compensation and estimated disturbance compensation term:$g^{-1}[-f - \hat{d} + \dot{\eta}]$.

This term $\hat{d}$helps compensate for the effect of the actual disturbance$d$.

Linear feedback term: $g^{-1}Ks$ helps improve transient response.

Switching (robustifying) term: $g^{-1}Q\,sgn(s)$. This component ensures the system's robustness against the disturbance estimation error $e_d = d - \hat{d}$ and other uncompensated uncertainties.

3.3. Stability Analysis Sketch

The authors use the Lyapunov method to analyze the stability of the closed-loop system comprising the robot, the SMC-DO controller eq. (12), and the disturbance observer from formulas *eq. (3)- (6)*.

Choose a composite Lyapunov function including both the sliding surface error and the disturbance estimation error:

$$V(s, e_d) = \frac{1}{2}s^T s + \frac{1}{2}e_d^T L^{-1} e_d \quad (13)$$

Calculate the derivative of *V*:

$$\dot{V}(s, e_d) = s^T \dot{s} + e_d^T L^{-1} \dot{e}_d \quad (14)$$

From *eq. (8)* and *(9)*, the dynamics of s under the SMC-DO control law: $\dot{s} = f + gu + d - \dot{\eta}_d$
substituting *u* from *equation (12)* gives *eq. (15)*.

$$\begin{aligned}\dot{s} &= f + g(g^{-1}[-f - \hat{d} + \dot{\eta}_d - Ks\text{-}Q\,sgn(s)]) + d - \dot{\eta}_d \\ &= (d - \hat{d}) - Ks\text{-}Q\,sgn(s) \\ &= e_d - Ks\text{-}Q\,sgn(s)\end{aligned} \quad (15)$$

From *eq. (6)*, we have: $\dot{e}_d(t) = -Le_d(t) - \dot{d}(t)$

Substituting into *eq. (14)* yields:

$$\begin{aligned}\dot{V} &= s^T(e_d - Ks - Q\,sgn(s)) + e_d^T L^{-1}(-Le_d - \dot{d}) \\ &= s^T e_d - s^T Ks - s^T Q\,sgn(s) - e_d^T e_d + e_d^T L^{-1}\dot{d} \\ &= -s^T Ks - Q\|s\|_1 - \|e_d\|^2 + s^T e_d + e_d^T L^{-1}\dot{d}\end{aligned} \quad (16)$$

where: $s^T Ks$ is the stabilizing term; $Q\|s\|_1 = s^T Q\,sgn(s)$ ensures robustness; $\|e_d\|^2 = e_d^T e_d$is the term reducing the estimation error e_d.

Stability Analysis:

- If $\dot{d} = 0$ (constant disturbance): $\dot{V} = -s^T Ks - Q\|s\|_1 - \|e_d\|^2 \leq 0 \rightarrow$ system is asymptotically stable.

- If $\dot{d} \neq 0$ (varying disturbance): Using the Cauchy-Schwarz inequality:

$$e_d^T L^{-1}\dot{d} \leq \|e_d\| L^{-1}\dot{d} \leq \|e_d\|^2 + \frac{1}{2}\left\|L^{-1}\dot{d}\right\|^2$$

Then: $\dot{V} \leq -s^T Ks - Q\|s\|_1 - \frac{1}{2}\|e_d\|^2 + \frac{1}{2}\left\|L^{-1}\dot{d}\right\|^2$

- If $\dot{d}$ is bounded, the system is stable.

4. SIMULATION RESULTS

4.1. Simulation Setup

A robot model with parameters given in Table 1 is used, following a reference trajectory:$x_d = 2 + 2\cos 0.4 * t; y_d = 2 + 2\sin 0.4 * t$ passing through regions with different terrains. The mixed terrain consists of flat terrain, slightly rough terrain, and very rough terrain, defined by disturbance signals from the ground to the robot's wheels.

Table 1. Robot Parameters for Simulation

Parameter	Value	Unit
Robot Mass	15	kg
Wheel Radius	0.03	m
Distance between wheels	0.05	m
Robot Moment of Inertia	0.27	kg.m^2
Wheel Moment of Inertia	0.0075	kg.m^2

To simulate the robot tracking the trajectory on mixed terrain, the paper compares the following controllers: PID controller (optimally tuned); Traditional SMC controller (without NDO, with a sufficiently large gain K); against the proposed SMC-NDO controller based on quality indices. Controller parameters are presented in *table 2*.

Table 2. Controller Parameters

Controller	λ	K	Φ	L
SMC	25	12	0.1	none
SMC+NDO	25	12	0.1	50

4.2. Simulation Results

The simulation results for the mobile robot tracking the set trajectory are described in *figures 3 to 11*.

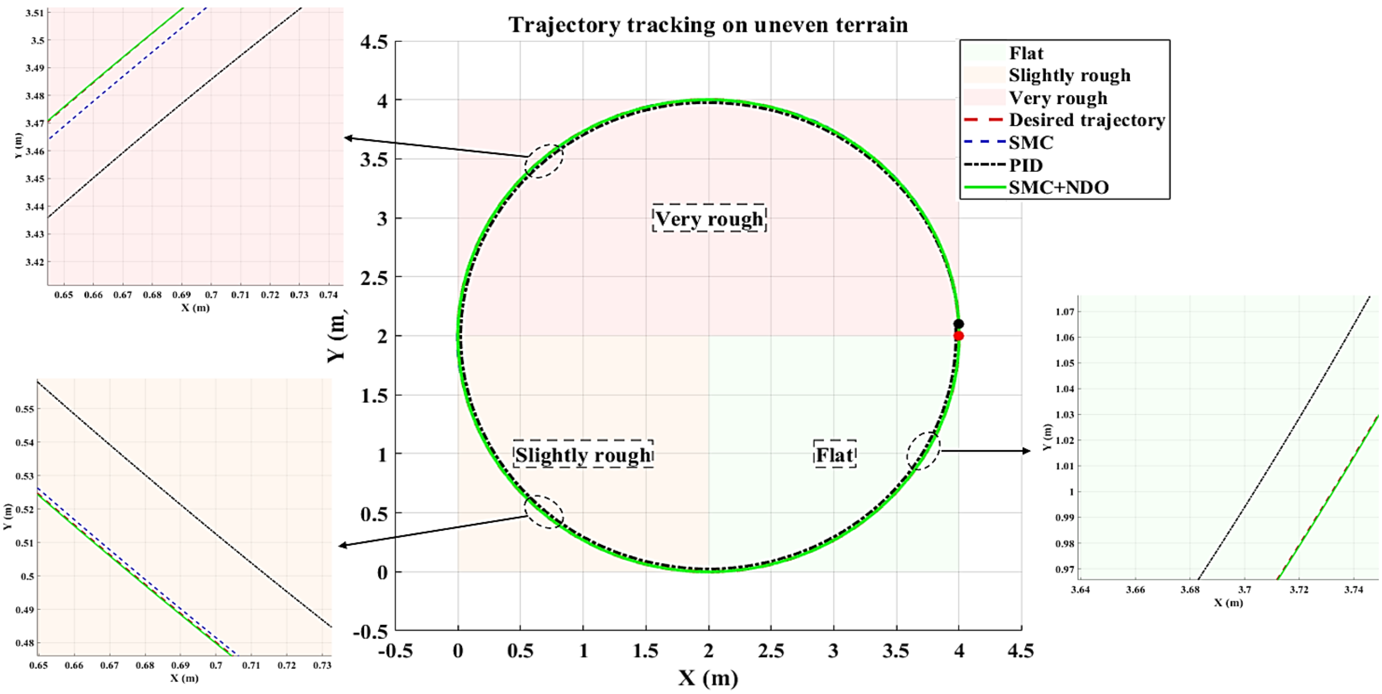

Figure 3. Trajectory tracking on uneven terrain

The graph compares the circular trajectory tracking performance of three controllers: SMC (dotted blue), PID (dash-dot balck), and SMC+NDO (solid green) across terrains ranging from flat to very rough. The zoomed-in section in the top-left provides details of the initial tracking phase, starting around coordinates (0m, 4m). Here, the superiority of SMC+NDO is clearly visible: while SMC and especially PID show significant deviation from the desired trajectory (dashed red) initially, SMC+NDO follows the path almost immediately. This rapid and accurate response demonstrates the effectiveness of the NDO in quickly compensating for uncertainties and disturbances, confirming the advantage of SMC+NDO in terms of accuracy and convergence speed.

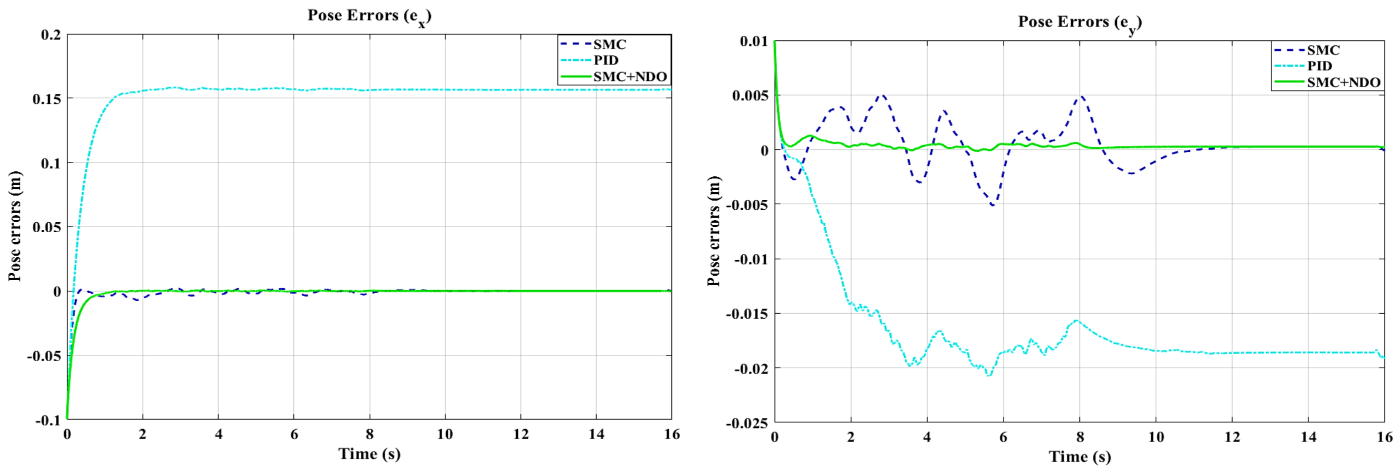

Figure 4. Pose errors – e_x, e_y

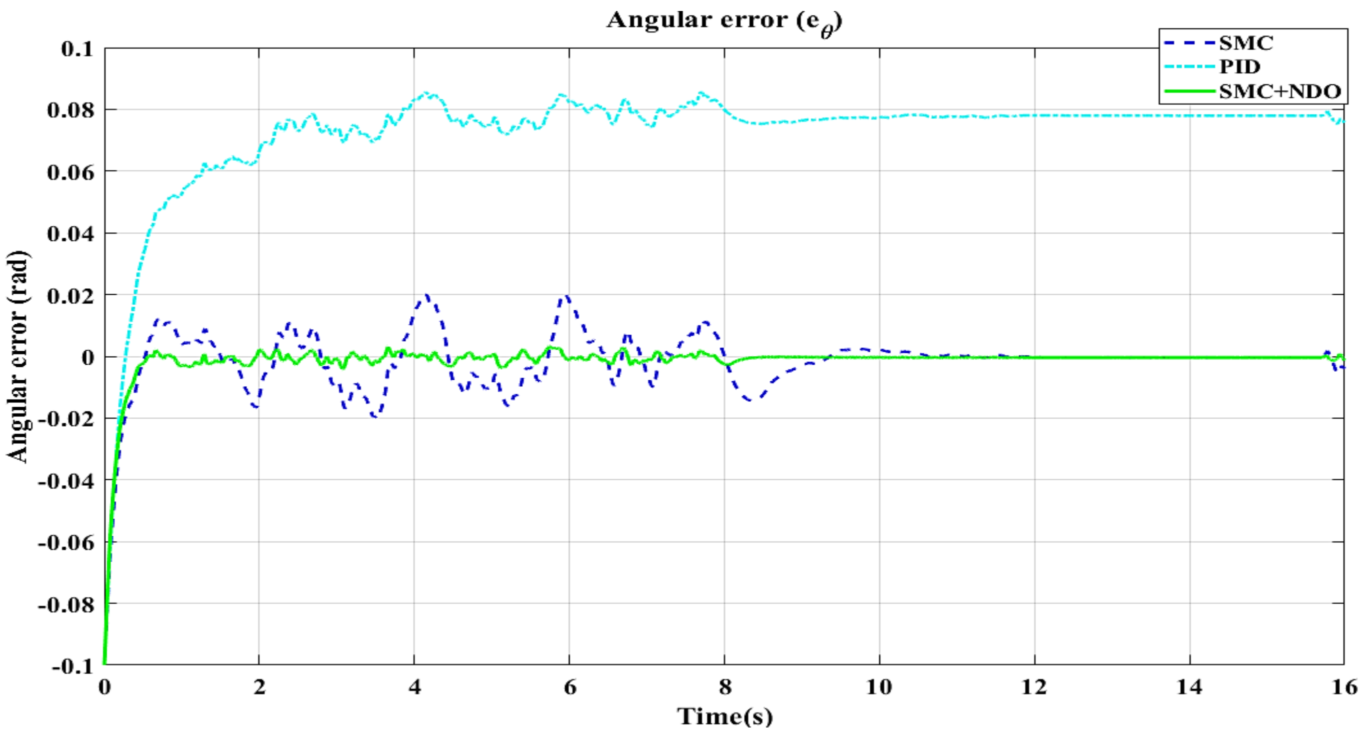

Figure 5. Angular error

Figure 4, 5 details the position error along the X-axis (e_x), Y-axis (e_y), and the angular error (e_θ) for the three controllers: SMC, PID, and SMC+NDO. The PID controller (dash-dot cyan) shows the largest errors, converging slowly towards 0.16m in e_x, $e_\theta$stabilizing around 0.075 rad in , and $e_y$oscillating around -0.02m in after the transient phase. The SMC controller (dashed blue) has a near-zero average error but exhibits significant oscillations, particularly in$e_y$ande_θ. In contrast, SMC+NDO (solid green) demonstrates superior performance, quickly bringing all three errors close to zero and maintaining high stability with very small oscillations, despite terrain changes.

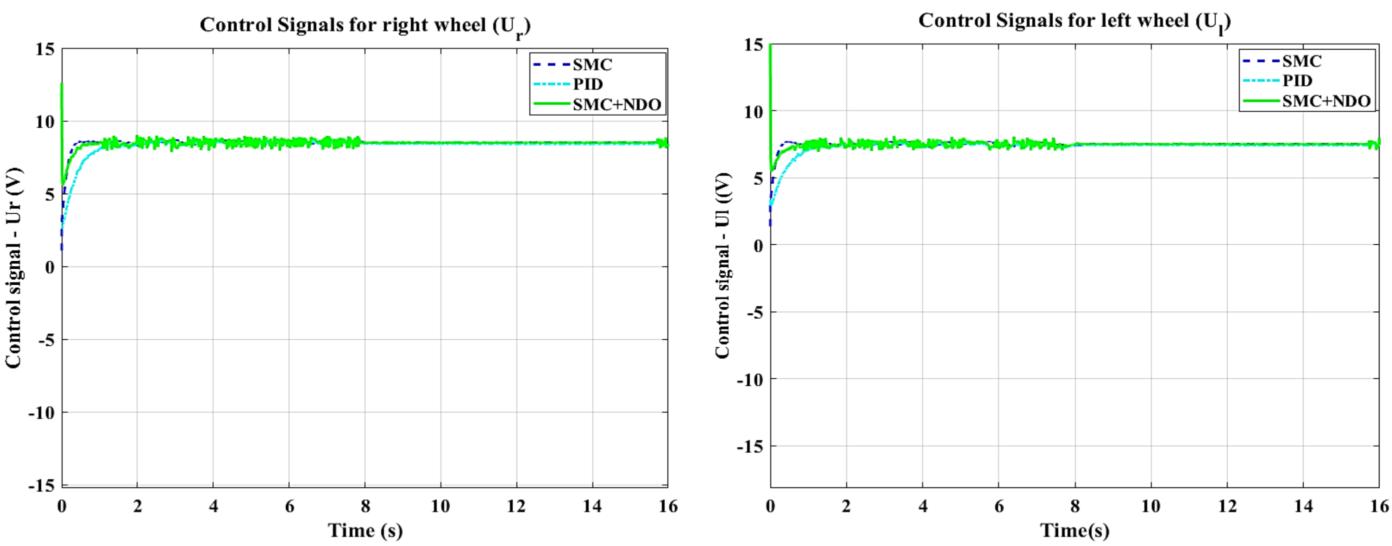

Figure 6. Control signal for left, right wheel

Figure 6 shows the control signals for the right and left wheels. Based on the provided graphs, the prominent advantage of the SMC+NDO method (green line) compared to the standard SMC (dashed blue line) is the **significant reduction of the high-frequency oscillation phenomenon (chattering)** in the control signal. While the SMC line exhibits distinct oscillations around the steady value, the SMC+NDO line is considerably smoother, indicating that the NDO worked effectively in estimating and compensating for disturbances, reducing the discontinuity of the control signal inherent in SMC. Although the PID signal (cyan line) is also relatively smooth, SMC+NDO inherits the robustness of SMC while improving the control signal quality, as demonstrated by this reduction in oscillation.

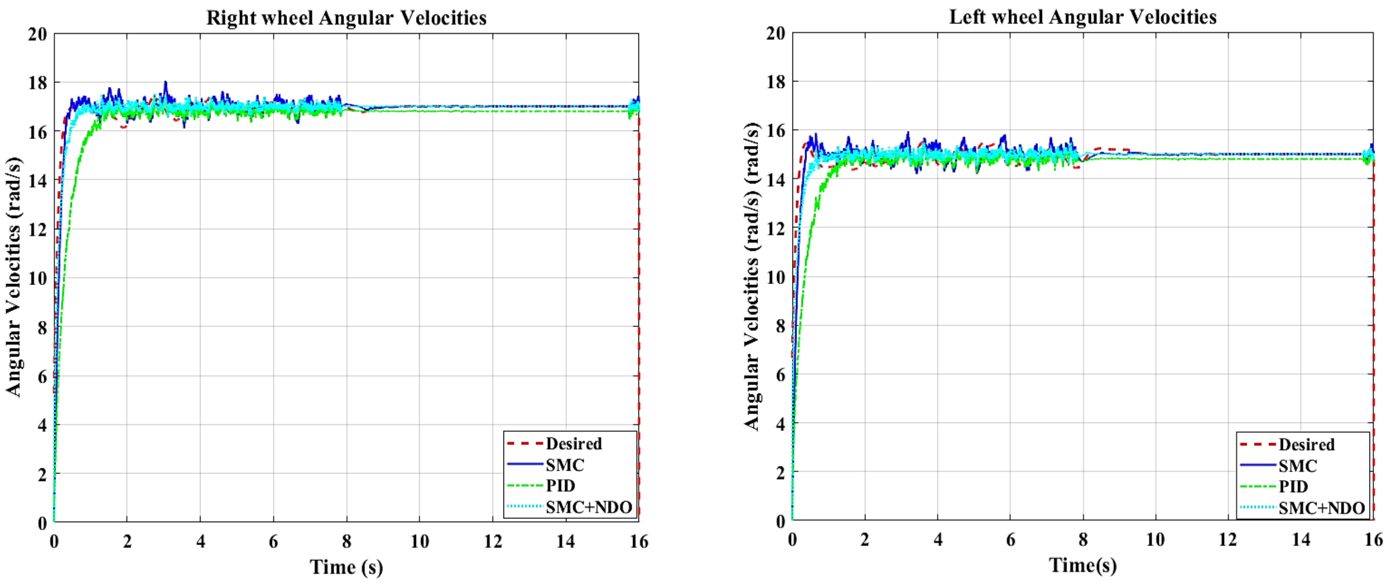

Figure 7. Left, right wheel angular velocities

Figure 7 depicts the angular velocities of the robot's right and left wheels. The PID controller (dashed green line) shows a slower response time and significant chattering around the desired value of 16 rad/s. The SMC controller (solid blue line) responds faster but also clearly exhibits chattering. Most notably, the SMC+NDO controller (dotted cyan line) reaches the desired velocity of 16 rad/s very quickly and maintains this value stably with significantly less oscillation than the other two controllers. This stability is well maintained across different terrain regions ("Very rough", "Slightly rough", "Flat"), demonstrating the precise and robust velocity control capability of SMC+NDO.

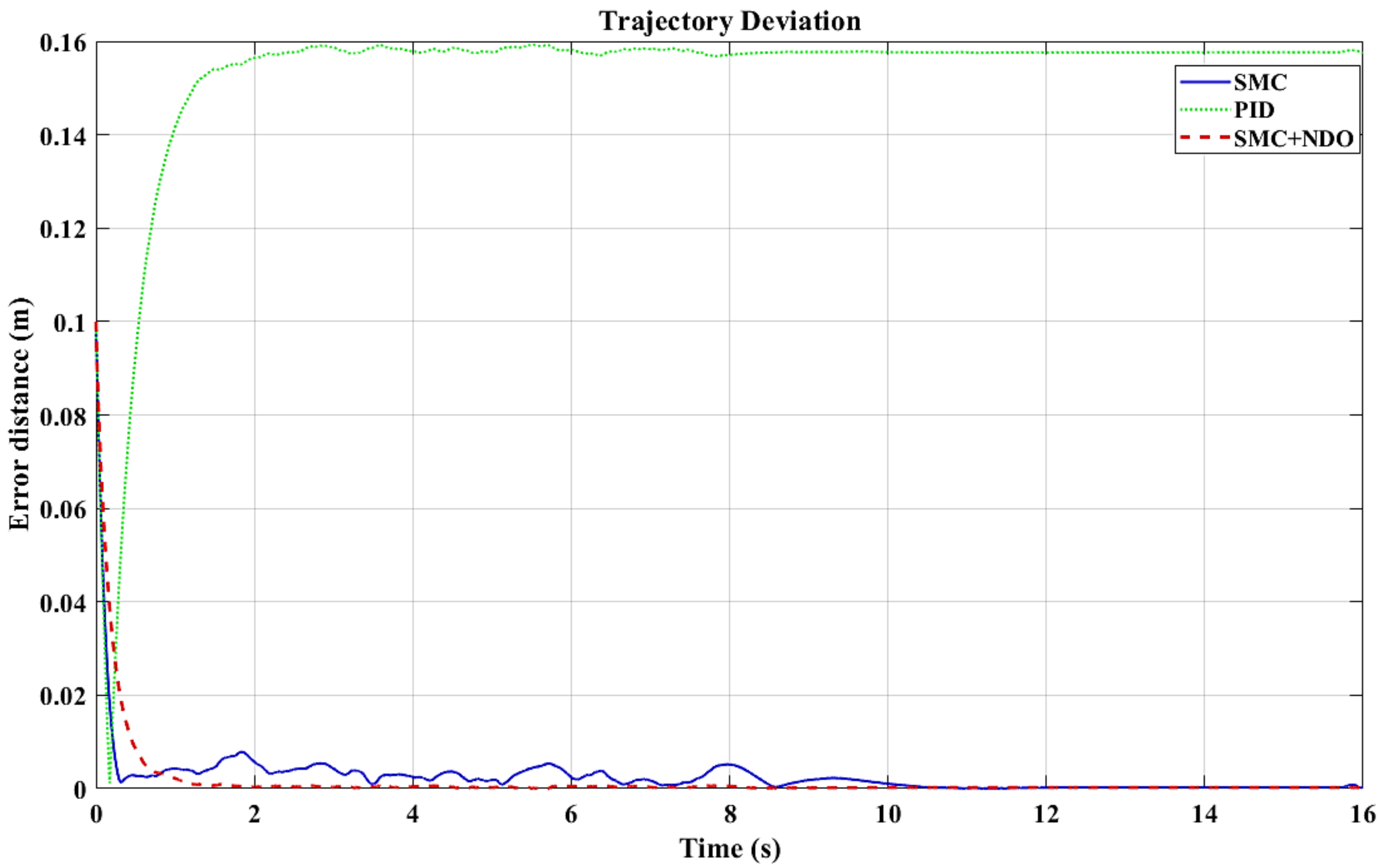

Figure 8. Trajectory deviation

Figure 8 shows the trajectory deviation distance for the three controllers: SMC, PID, and SMC+NDO. The PID controller (green) exhibits a very large deviation, initially spiking and quickly settling at a high level of about 0.158 meters throughout the simulation. The SMC controller (blue) significantly reduces the deviation compared to PID, keeping the distance error below 0.008 meters after the initial transient phase, but still shows minor oscillations. Superior to both, the SMC+NDO controller (cyan) almost instantaneously minimizes the deviation distance to nearly zero (≈ 0m) and maintains this accuracy stably across all terrain types.

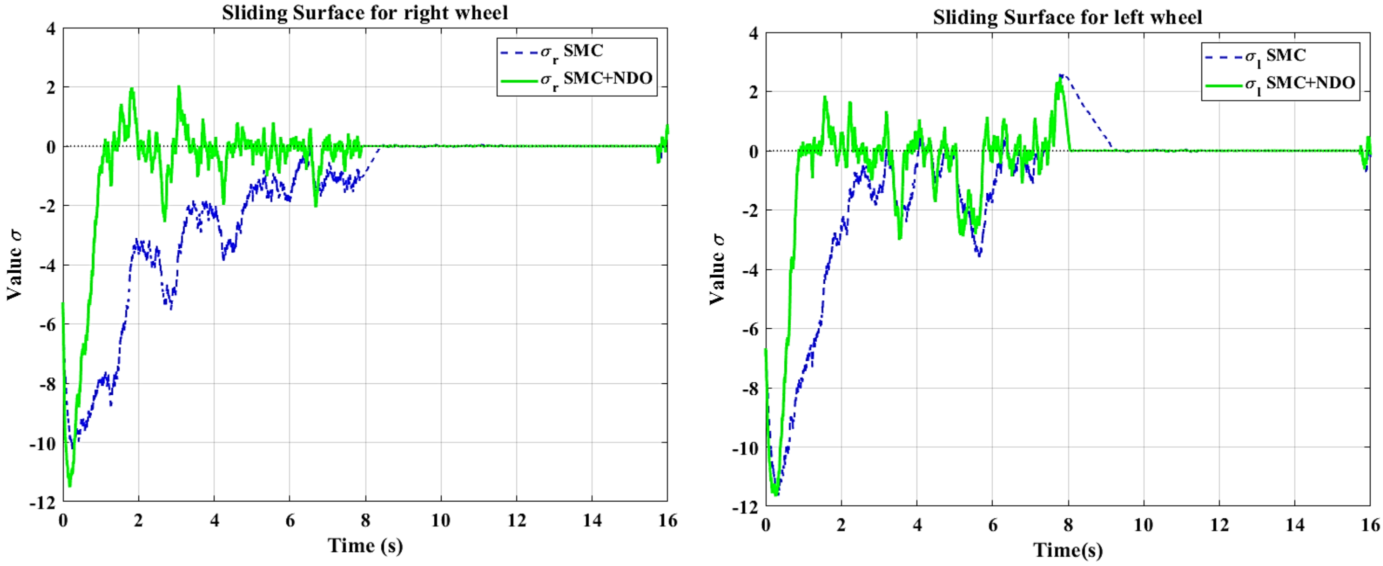

Figure 9. Sliding surface for left and right wheel

Figure 9 plots the sliding surface value (value σ) over time for the robot's right and left wheels. It compares the standard SMC controller (dashed blue) and the SMC+NDO (solid line green). The goal of sliding mode control is to drive σ to zero and keep it there. The standard SMC shows *σ* converging very slowly to zero, taking about 8-9 seconds, and exhibiting large oscillations, especially during the "Very rough" terrain phase. Conversely, the SMC+NDO controller demonstrates clear superiority by driving σ to zero extremely quickly, within 0.5-1 second for both wheels. After convergence, SMC+NDO maintains σ stably near zero with very small oscillations, regardless of terrain changes, proving significantly higher convergence speed and robustness.

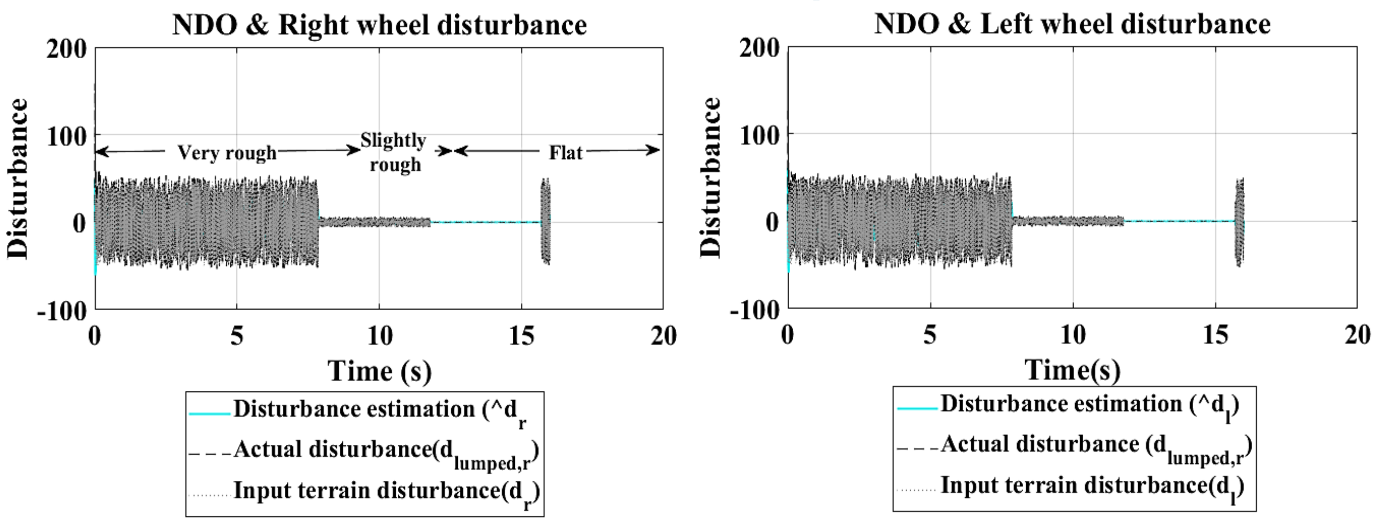

Figure 10. NDO, left and right wheel disturbance

Figure 10 shows that the Nonlinear Disturbance Observer (NDO) estimate (dotted cyan) closely tracks the actual disturbance from the terrain (solid gray) for both wheels. The NDO successfully tracks both large amplitude disturbances (around +/- 50) on rough terrain and smaller disturbances on flatter terrain.

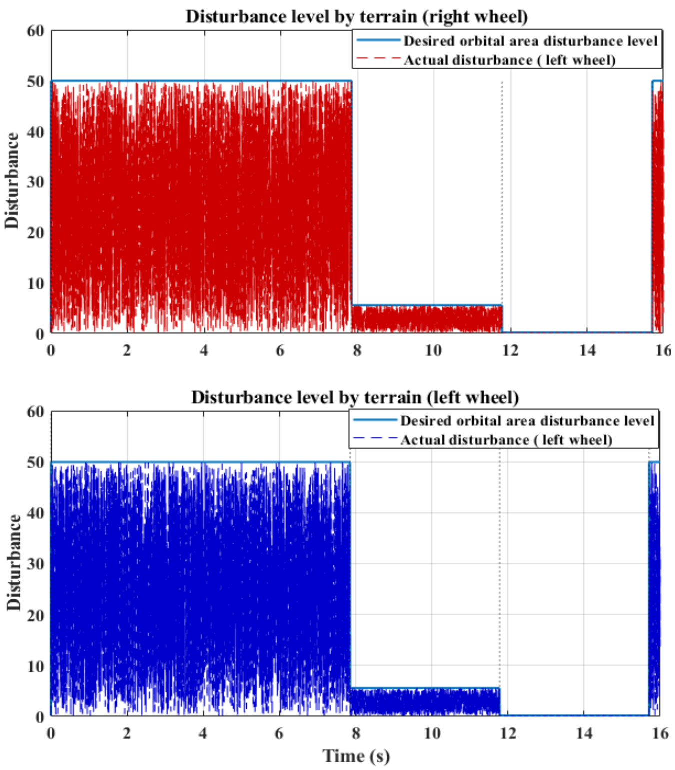

Figure 11. Disturbance level by terrain

Figure 11 displays the disturbance level over time. The top graph shows the disturbance acting on the right wheel (red plot), while the bottom graph depicts it for the left wheel (blue plot). This signal is intended to simulate the effect of changing terrain conditions on the robot's wheels.

5. CONCLUSIONS

This paper addressed the challenging problem of trajectory tracking for mobile robots operating in mixed terrain environments, where varying friction, model uncertainties, and external disturbances significantly degrade performance. To overcome these challenges, an integrated control strategy combining Sliding Mode Control (SMC) with a Nonlinear Disturbance Observer (NDO) was proposed and evaluated. The NDO was designed to effectively estimate the lumped disturbances acting on the robot, including the complex effects arising from changing terrain conditions.

The core contribution of this work lies in the utilization of the real-time disturbance estimation provided by the NDO for active compensation within the SMC framework. This approach allows for a significant reduction in the magnitude of the SMC switching gain, thereby effectively mitigating the detrimental chattering phenomenon commonly associated with traditional SMC, while simultaneously preserving the system's robustness.

The effectiveness of the proposed SMC-NDO controller was validated through comprehensive simulations. A mobile robot model was tasked with tracking a reference trajectory across simulated mixed terrains encompassing flat, slightly rough, and very rough conditions. Comparative results clearly demonstrated the superiority of the proposed SMC-NDO controller over both a traditional SMC controller and a standard PID controller. Specifically, the SMC-NDO controller achieved significantly higher trajectory tracking accuracy, exhibited much faster convergence of tracking errors, and drastically reduced chattering in the control signals and wheel velocities. Furthermore, the simulation results confirmed the NDO's capability to accurately estimate the time-varying disturbances generated by the mixed terrain.

In summary, the proposed SMC-NDO control strategy offers a robust and effective solution for enhancing the trajectory tracking performance of mobile robots in complex, mixed-terrain environments. By intelligently estimating and compensating for disturbances, the controller successfully achieves high accuracy and robustness while significantly suppressing chattering. Future work could involve experimental validation on a physical mobile robot platform and investigation into adaptive tuning mechanisms for the observer and controller gains to further enhance performance under varying operating conditions.

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