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Design of Subtractive Fuzzy Clustering-Based PI Controller for Level Control of Quadruple Tank System with Dead Time

Mahammedrafi. G[1], R. Dhanalakshmi[2], and Rambabu Busi[3]

*[*1]Research Scholar, Department of Electronics and Instrumentation Engineering, Annamalai University, Tamil Nadu, India; Email: ramahammed@gmail.com*
[2]Department of Electronics and Communication Engineering, Thanthai Periyar Government Institute of Technology, Vellore-02, Tamil Nadu, India; Email: dhanavishnu02@gmail.com
[3]Department of Electronics and Communication Engineering, Lakireddy Bali Reddy College of Engineering(A), Mylavaram, India; Email: rams1315@gmail.com

***Correspondence:** *Mahammedrafi. G, ramahammed@gmail.com*

ABSTRACT- The performances of the Proportional-Integral controller, Fuzzy Logic controller, and Subtractive Fuzzy Clustering-based PI controller (SFC-PI) have been investigated for regulating the level in a Quadruple Tank System with Dead Time (QTSWDT). The QTSWDT is an ideal benchmark to test various control approaches since it has nonlinear dynamics and complicated interactions between tanks. While classic PI controllers are successful in controlling linear systems, they face difficulties in regulating the nonlinearities and cross-couplings inherent in QTSWDT. Fuzzy logic controllers offer extended adaptation to nonlinearities but require substantial tuning. The SFC-PI controller, which offers subtractive fuzzy clustering to instinctively generate fuzzy rules, surpasses the other techniques by significantly reducing ISE, IAE, settling time, and peak overshoot. Simulation outputs disclose that the SFC-PI controller has the best overall performance, making it a competent choice for complex nonlinear control applications.

Keywords: Quadruple Tank System with Dead Time (QTSWDT), Level Control, PI Controller, Nonlinear and Subtractive Fuzzy Clustering.

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1. INTRODUCTION

Due to the nonlinear dynamics and interaction between the tanks, level regulation in the Quadruple Tank System with Dead Time is a challenging problem. These systems are usually employed in industrial applications such as food manufacturing, chemical processing, and water treatment, where accurate and precise control of fluid levels is needed. In a four-tank system, the level in each tank depends not just on the inflow and outflow rates but also on the flow rates among the tanks, which leads to intricate interconnected dynamics [1]. Hence, attaining consistent and precise level regulation requires sophisticated control methods that can competently manage interactions, nonlinearity, and disturbances. For regulating levels in four-tank systems, different regulation methods have been recommended and executed. These techniques range from classic linear controllers to more sophisticated and adaptable methods, each having its advantages and disadvantages.

In industrial applications the most basic and commonly employed control method is PI controller. It merges a proportional term that addresses the present error with an integral term that removes steady state error. The PI controller has been widely applied in linear systems, where it provides a balanced trade-off between system stability and performance. Alvarado et.al. (2006) [2] showed that PI controllers are capable of efficient for specific linearized configurations of the system. Nonetheless, the functionality of PI controllers diminishes when confronted with of nonlinear interactions and interdependencies between tanks, which are intrinsic to the four-tank configuration. Although PI controllers are simple to create and execute, they frequently lack adequacy for the precise management of intricate systems such as the four tank system requiring minimal tuning.

As an alternative approach for managing nonlinear systems, the Fuzzy Logic Controller has gained growing recognition. Fuzzy logic, a type of reasoning that accommodates uncertain or vague information, is employed by FLCs. In this case, fuzzy rules are applied to correlate input variables with control actions. FLCs do not require an exact mathematical representation of the system, making them especially suitable for systems with considerable unpredictability and difficulty. Research conducted by Lee and Kim (2016) showed that fuzzy logic controllers applied to a four-tank system significantly decreased overshoot and enhanced transient response compared to conventional PI controllers. However, FLCs need rules, which can be complicated, and defining appropriate membership functions can be challenging. Furthermore, in certain situations, tuning these parameters manually might still lead to less-than-ideal performance [3-4].

Particularly for controlling multivariable systems such as the four-tank system, Model Predictive Control (MPC) has risen as an effective approach. A system model to forecast future behavior and intensify control inputs across a limited timeframe is employed by MPC [5]. It is ideal for systems that require many interrelated variables and limitations. For the four-tank system, Nonlinear Model Predictive Control has been the focus of recent studies because it manages the intrinsic nonlinearities more effectively. While ensuring stability and performance, a nonlinear MPC to address input and state limitations has been suggested by Zheng, A. (1997) [6]. The primary advantage of MPC lies in its capacity to anticipate upcoming disturbances and change control inputs accordingly, rendering it particularly robust in changing environments. However, its computational complexity continues to pose a challenge for real-time execution, especially in extensive systems.

Fuzzy logic control techniques with adaptive adjustment of the integral and proportional gains are merged by the Adaptive PI Fuzzy Controller. The PI gains are flexibly changed by this approach according to the system's response and render better performance compared to conventional PI controllers. It combines the stability offered by PI control with the adaptability of fuzzy logic. A Neuro-Fuzzy controller for the four-tank system is proposed, and a decrease in steady-state error and overshoot in contrast to conventional PI control is manifested [7]. Its flexible characteristics allow real-time modifications of the controller parameters, improving robustness against disturbances and abnormalities in the system. The difficulty resides in choosing suitable fuzzy rules and membership functions, even though this approach improves performance.

For nonlinear and intricate systems, including the four-tank system, Artificial Neural Networks have been popular in control methods. Without depending on exact mathematical models and by using learning algorithms, ANNs can control systems that exhibit complex dynamics. Machine learning-based models have been proposed for the Quadruple Tank System [8]. The input-output correlations presented in the Neural Network PI controller (NN-PI) predict suitable PI controller gains by allowing the network to learn. Throughout different operating conditions, this controller enhances control efficiency. However, a major obstacle in utilizing neural networks is the need for substantial datasets for training and the estimated requirements related to real-time monitoring [9].

Reinforcement Learning (RL) allows systems to find the best control plans through interaction with their environment. Because of RL's capacity to learn from experiences and enhance over time, it has been widely recognized in process control, especially in the level control of four-tank systems. By utilizing Q-learning to improve the control strategy in real time, RL has been implemented in a four-tank setup [10]. By obtaining feedback from the system's performance, the RL controller adjusts and improves control actions through many iterations. The principal advantage of this method is its capacity to improve the control approach without needing an exact system model. Even though it has advantages, RL-based control faces challenges such as necessitating significant training and the possibility of gradual convergence when used for real-time control.

In order to address the restrictions of conventional fuzzy logic controllers, the Subtractive Fuzzy Clustering-Based PI Controller (SEC-PI) is designed by automatically generating fuzzy rules via a clustering process. Subtractive clustering is a technique used to partition the input space into groups, from which fuzzy rules can be derived and membership functions are established [11]. This technique improves the adaptability of fuzzy logic systems to fluctuating system conditions and eliminates the need for manually creating fuzzy rules. This paper investigates the control strategies, namely conventional Proportional-Integral (PI) controller, Fuzzy Logic Controllers (FLC), and recent techniques like Subtractive Fuzzy Clustering-Based PI controllers (SFC-PI). This study represents a Subtractive Fuzzy Clustering-Based PI controller designed for a nonlinear Quadruple Tank System with Dead Time process.

A novel Deep Neural Fuzzy based Fractional Order PID (DN-FFOPID) controller has been introduced for managing levels in nonlinear quadruple tank systems. The suggested controller combines deep neural networks, fuzzy logic, and fractional-order PID control to enhance tracking performance and disturbance rejection in multivariable tank systems. A six-layer deep neural network was utilized to enhance controller performance and minimize steady-state error, overshoot, and settling time. Nonetheless, the suggested DN-FFOPID controller has significant computational complexity and demands thorough training of neural network parameters [12]. To enhance robustness, disturbance rejection, and tracking precision under nonlinear operating conditions, an optimal hybrid Fractional-Order Type-2 Fuzzy PID (FO-T2FPID) controller for the nonlinear quadruple tank system has been introduced and validated through Hardware-in-the-Loop co-simulation utilizing the dSPACE platform. The suggested control strategy has been executed in a real-time setting utilizing MATLAB/Simulink and dSPACE hardware to confirm practical viability. Despite the FO-T2FPID controller demonstrating enhanced control performance, the approach entails significant computational complexity and advanced hardware implementation needs [13]. A sophisticated control approach is suggested for the nonlinear quadruple tank system by combining Model Predictive Control (MPC) with an Adaptive Neuro-Fuzzy Inference System (ANFIS). The suggested framework integrated Linear MPC (LMPC) and Nonlinear MPC (NMPC) with an ANFIS-based observer to improve state estimation. Moreover, Particle Swarm Optimization (PSO) was used to optimize the ANFIS parameters to enhance modeling precision and flexibility [14].

The suggested approach varies from earlier published Fuzzy-PI and SFC-PI methods in several significant ways. Initially, the suggested controller is specifically designed for a

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nonlinear interacting QTSwDT process that encompasses dead-time features and significant loop interactions. Secondly, the suggested Subtractive Fuzzy Clustering-based PI controller automatically derives fuzzy rules and membership functions from process data via subtractive clustering, thus removing the need for manual rule creation and minimizing controller design effort. Third, the suggested approach utilizes just six automatically created fuzzy rules, leading to a smaller rule base and decreased computational load compared to traditional Fuzzy-PI controllers. Fourth, the controller adjusts the proportional and integral gains in real-time based on process conditions, enhancing resilience to nonlinearities, disturbances, and changes in parameters.

2. MATHEMATICAL REPRESENTATION OF QUADRUPLE TANK SYSTEM WITH DEAD TIME

The Quadruple Rank System with Dead Time is a multi-variable, nonlinear process usually utilized for analyzing control strategies. It consists of four linked tanks, with the fluid level in each individual tank controlled by changing the inflow and outflow rates. The mathematical representation of this tank structure describes the dynamics related to fluid levels in terms of mass balances and the flow dynamics between tanks. *Figure 1* exhibits the symbolic representation of the Quadruple Tank System with dead time. The QTSwDT contains four pumps: Pump1, Pump2, Pump3, and Pump4, along with four tanks: Tank1, Tank2, Tank3, and Tank4. The main purpose of regulating QTSwDT is to control the water levels h_1 in Tank1 and h_2 in Tank2. The input control voltages are V_1 and V_2. The system displays interacting multivariable dynamics because each of the pumps impacts both of the outputs. Unmeasured disturbances can be introduced by extracting water from the upper tanks and transferring it to the lower reservoir. This introduces the researcher to disturbance rejection and reference tracking. The objective of control is to keep the liquid levels in the two lower tanks at the specified levels. The water discharged from Tank3 and Tank4 is collected by Tank1 and Tank2, which are positioned directly beneath Tank3 and Tank4. The water that exits Tank1 and Tank2 is gathered in the reservoir tank located at the bottom. The independent dead times are achieved by adding two extra pumps in place of the two valves found in the original QTP. Control input voltages are bisected between pumps to give n_i (i=1,...4), the input voltage to pump I, delayed at time t by the dead times, as given in the following *equation (1)*.

$$\left.\begin{aligned} n_1 &= v_1(t-\Delta_1).\gamma_1 \\ n_4 &= v_1(t-\Delta_4).(1-\gamma_1) \\ n_2 &= v_2(t-\Delta_2).\gamma_2 \\ n_3 &= v_2(t-\Delta_3).(1-\gamma_2) \end{aligned}\right\} \qquad (1)$$

This updated setup allows for the application of four separate Dead Times Δj(i=1.2….4) between inputs and outputs.

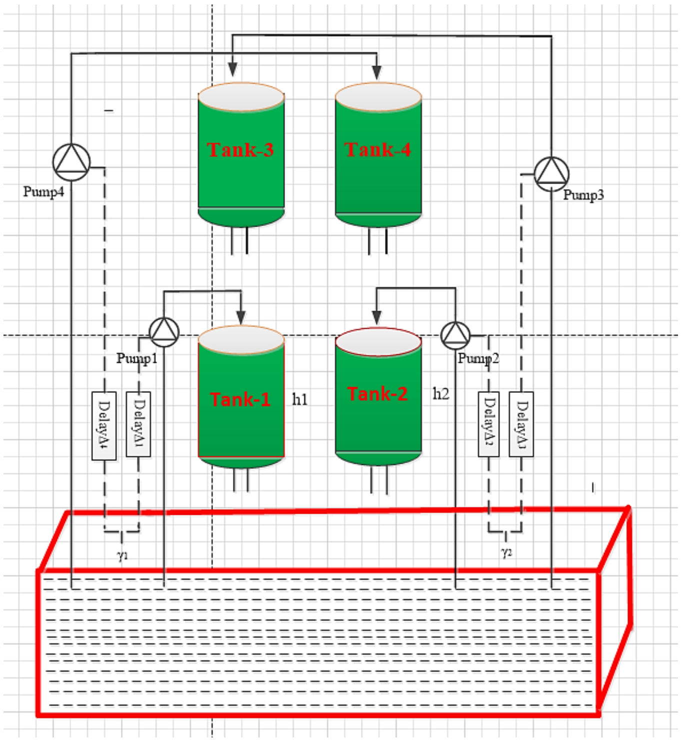

Figure 1. Quadruple Tank System with dead time

The dynamics are derived by employing the mass balance equation and Bernoulli's principle. The non-linear model established from *equations (2)* through *eq. (5)* is utilized for control development [15]. The non-linear modeling of QTSwDT is presented as follows.

$$\dot{h}_1 = \frac{(\gamma_1 k_1 u_1(t-\Delta_1)k_{12})}{A_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \left(\frac{-a_1}{A_1}\right)\sqrt{2gh_1} \qquad (2)$$

$$\dot{h}_2 = \frac{(\gamma_2 k_2 u_2(t-\Delta_2)k_{22})}{A_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \left(\frac{-a_2}{A_2}\right)\sqrt{2gh_2} \qquad (3)$$

$$\dot{h}_3 = \frac{((1-\gamma_2)k_3 u_2(t-\Delta_3)k_{32})}{A_3} + \left(\frac{-a_3}{A_3}\right)\sqrt{2gh_3} \qquad (4)$$

$$\dot{h}_4 = \frac{((1-\gamma_1)k_4 u_1(t-\Delta_4)k_{42})}{A_4} + \left(\frac{-a_4}{A_4}\right)\sqrt{2gh_4} \qquad (5)$$

Where Ai represents the cross-section of tank i; a_i denotes the cross-section of the outlet hole in tank i; hi indicates the water level in tank i; and g is the acceleration due to gravity. K_j and K_{j2} are constants with pump j. The nonlinear model is reduced to a linearized state space model under balanced conditions. The linearized state space equations are given as follows:

$$\frac{d\hat{v}(t)}{dt} = \sum_{i=1}^{4} E_i\,.\,\hat{u}(t-\Delta_i) + D.\hat{v}(t) \qquad (6)$$

$$\hat{Z}(t) = F.\hat{v}(t) \qquad (7)$$

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Where $D=\begin{pmatrix} \frac{-1}{L_1} & 0 & \frac{A_3}{A_1L_3} & 0 \\ 0 & \frac{-1}{L_2} & 0 & \frac{A_4}{A_2L_4} \\ 0 & 0 & \frac{-1}{L_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{L_4} \end{pmatrix}$

$$E_1=\begin{pmatrix} \frac{\gamma_1 K_1}{A_1} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, E_2=\begin{pmatrix} 0 & 0 \\ 0 & \frac{\gamma_2 K_2}{A_2} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, E_3=\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{(1\text{-}\gamma_2)K3}{A_2} \end{pmatrix}$$

$$E_4=\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{(1\text{-}\gamma_1)K_4}{A_4} & 0 \end{pmatrix}$$

$$E=E_1+E_2+E_3+E_4 E=\begin{pmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1\text{-}\gamma_2)k_3}{A_3} \\ \frac{(1\text{-}\gamma_1)k_4}{A_4} & 0 \end{pmatrix} \quad (8)$$

$$F=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} L_i=\sqrt{\frac{2.h_i^0}{g}}.\frac{A_i}{a_i} \quad (9)$$

Where h_i^0 are steady state values of h_i.

The chain matrix is given by $\hat{z}(s)=Q(s).\hat{u}(s)$

Where,

$$Q(s)=\begin{bmatrix} Q_{11}(s) & Q_{12}(s) \\ Q_{21}(s) & Q_{22}(s) \end{bmatrix}$$

$$=\begin{bmatrix} \frac{(\gamma_1 L_1 u_1)e^{-s\Delta_1}}{(E_1)(1+sL_1)} & \frac{(1\text{-}\gamma_2)L_1 u_3 e^{-s\Delta_3}}{(B_1)(1+sL_3)(1+sL_1)} \\ \frac{(1\text{-}\gamma_1)L_2 u_4 e^{-s\Delta_4}}{(B_2)(1+sL_4)(1+sL_2)} & \frac{(\gamma_2 L_2 u_2)e^{-s\Delta_2}}{(B_2)(1+sL_2)} \end{bmatrix}$$

Quadruple tank system with dead time can be analyzed at the operating point of minimal stage characteristic. This characteristic depends on the values of γ_1 and γ_2. For minimal stage, $\gamma_1+\gamma_2 \in (1, 2)$. The values of γ_2 and γ_1 are chosen as 0.60 and 0.65. The Quadruple tank system with dead time parametric variables is outlined in *table 1*. The four pumps in this system have control input voltages ranging from [3, 12] volts. The height of the individual tank is 23 cm.

3. FUNCTIONAL DIAGRAM OF SUBTRACTIVE FUZZY PI CONTROLLER

The functional diagram of the subtractive fuzzy PI controller is shown in *figure 2*. Four subtractive fuzzy logic controllers are used to control the heights of the tanks h_1 and h_2 in Tank1 and Tank2. The error e_1 is the difference between SetpointTank1 and the height of Tank1 h_1. Similarly, error e_2 is the difference between SetpointTank2 and the height of Tank2 h_2. The input to subtractive fuzzy logic controller1 and subtractive fuzzy logic controller 2 is error e_1. Similarly, the input to subtractive fuzzy logic controller3 and subtractive fuzzy logic controller 4 is error e_2. The output of the the subtractive fuzzy logic controller 1 has gain *Kc*, which is multiplied by error $e_{1(t)}$ to generate *Kce1(t)*. The output of the subtractive fuzzy logic controller 2 is gain *Ki*, which is multiplied by the integral of error $e_{1(t)}$ to generate *Ki∫e1(t)*. These two terms are added to produce control input voltage u1. Similarly, the output of the subtractive fuzzy logic controller3 has gain *Kc*, which is multiplied by error e2(t) to generate *Kce2(t)*. The output of the subtractive fuzzy logic controller4 is gain *Ki*, which is multiplied by the integral of error $e_{2(t)}$ to generate *Ki∫e2(t)*. These two terms are added to produce control input voltage u_2. The Subtractive Fuzzy Clustering Controller uses a reduced rule base compared to the conventional FLC. The number of rules generated by this Subtractive Fuzzy Clustering Controller is six, which is low when compared to the conventional FLC. The Subtractive Fuzzy Clustering Controller automatically extracts fuzzy rules from input-output data and membership functions from cluster centers .

Table 1. Parametric variables of Quadruple tank System with Dead Time

Kj(cm^3/(s.V))	Kj2(cm^3/s)	j	Deadtime(Δ_j)	a_i(cm^2)	A_i(cm^2)
3.80	2.71	1	0.5	0.26	12
3.83	2.0	2	0.6	0.26	20
3.90	3.07	3	0.7	0.2	12
3.75	3.5	4	0.9	0.2	20

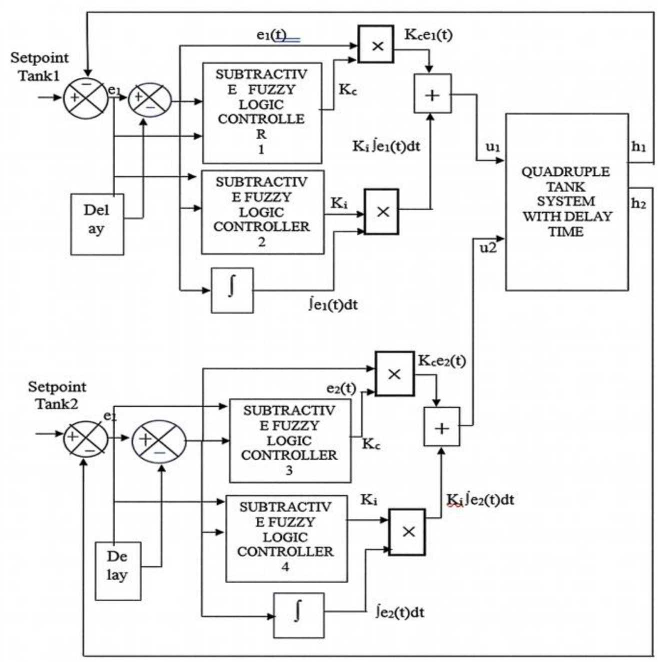

Figure 2. Functional diagram of Subtractive Fuzzy PI Controller

4. OPEN LOOP RESPONSE AND PI CONTROLLER DESIGN

The non-feedback retort of the quadruple tank system with dead time is obtained by applying step inputs to the control inputs u1 and u2. The resulting variations in the levels of Tank1, Tank2, Tank3, and Tank4 are recorded and presented in *figure 3*. The process parameters are determined using the two-point method (Bequette, B. W., 2010) [16].

$$\text{Time constant, } \upsilon = -1.5(t_1 - t_2) \quad (10)$$

$$\text{Dead time, } t_e = t_2 - \upsilon \quad (11)$$

$$\text{Process gain, Kp} = \frac{\text{change in output}}{\text{change in input}} \quad (12)$$

Time instant t_2 is determined at 63.2% of the final equilibrium value. Similarly, t_1 is obtained at 28.3% of the final equilibrium value (B). The PI controller is developed using the Ziegler-Nichols (Z-N) tuning technique to regulate the levels of Tank1 and Tank2 by adjusting the inflows u_1 and u_2, respectively. The Z-N tuning parameters are provided below:

$$\text{Controller gain, } K_c = \frac{0.9\upsilon}{t_e * K_p} \quad (13)$$

$$\text{Integral time, } T_i = 3.33 * t_e \quad (14)$$

The obtained process parameters and PI controller parameters are presented in *table 2*.

Table 2. Quadruple Tank System with Dead Time Process and PI controller Parameters

Tank	Process Gain (K_p)	Time Constant (υ)	Dead Time (t_e)	Proportional Gain (K_c)	Integral Time (T_i)	Integral Gain (K_i)
Tank1	2.48	9.534	0.446	7.758	1.485	5.224
Tank2	1.962	13.47	0.6485	9.53	2.1595	4.413

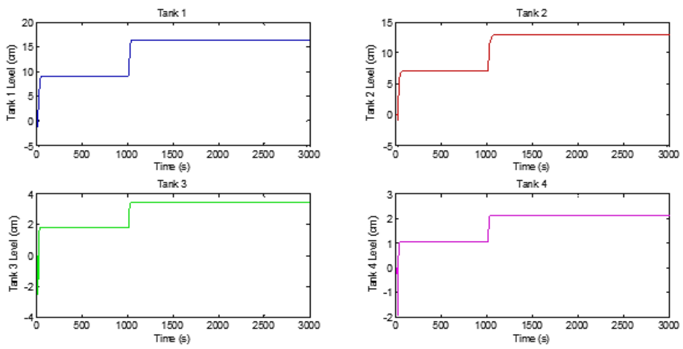

Figure 3. Open loop response of Quadruple tank System

The servo response of the quadruple tank process with a PI controller for setpoint changes—from 0 to 15 cm and from 15 to 20 cm in Tank1, and from 0 to 10 cm and from 10 to 15 cm in Tank2—is presented in *figure 4(a)*. The corresponding variation in the controller output is depicted in *figure 4(b)*. From these responses and *table 3*, it is observed that the PI controller produces overshoot, undershoot, and large ISE and IAE values. This necessitates the design of intelligent controllers, such as a fuzzy logic controller, for the nonlinear quadruple tank system.

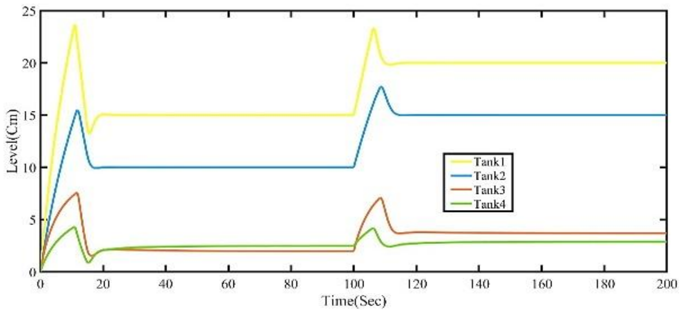

Figure 4(a). Process Output

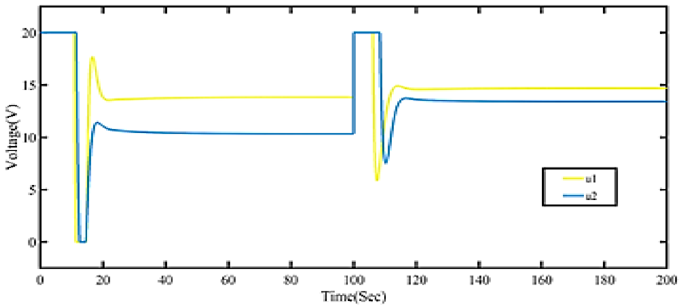

Figure 4(b). Controller Output
Figure 4. Servo response of quadruple tank process with dead time with PI controller

5. FUZZY LOGIC CONTROLLER

In numerous scientific and engineering fields FLCs have superseded traditional technologies because of their ability to mimic humans' approximate reasoning skills. The significance of fuzzy logic in control applications stems from its ability to define control issues through fuzzy linguistic variables and rules, providing the inherent benefit of nonlinear mapping, which results in enhanced controller performance. A fuzzy logic system can be viewed as a nonlinear transformation of an input data vector into an output vector, where this nonlinear relationship is characterized by linguistic terms calculated using numerical values. A fuzzy logic system stands out for its capability to process numerical data alongside linguistic information. The depth of this reasoning lies in the numerous possibilities that lead to various distinct mappings. Every fuzzy rule represents a statement in which the antecedent and the consequent include fuzzy propositions—statements that connect linguistic variables. Typically, the antecedent component of any rule is presented in a singular manner; however, the consequent of a fuzzy rule may adopt various forms, resulting in different fuzzy inference systems and consequently, various fuzzy schemes.

Table 3. Performance Measures of Quadruple Tank System with Dead Time with PI Controller

Setpoint Change	Tank1		Tank2	
	0 to 15 cm	15 to 20 cm	0 to 10 cm	10 to 15 cm
ISE	748.7	67.6	401.1	98.8

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IAE	88.94	21.16	70.02	34.18
ITAE	666.7	2106.3	582.1	3685.9
RMSE	2.501	1.843	1.866	1.473
Risetime(sec)	4.77	8.57	6.78	11.42
Settling time(sec)	23	16.83	28.99	27.63

A Mamdani-type fuzzy logic controller is designed to regulate the water levels h_1 in Tank1 and h_2 in Tank2. The inputs to the fuzzy logic controller are the error (e) and the rate of error (Δe), and the output is the water level in the tank. Gaussian membership functions are chosen for both the inputs and the output. The linguistic variables for the inputs and output are Negative Large (NL), Negative Small (NS), Zero (ZE), Positive Small (PS), and Positive Large (PL). The rule table for this controller is shown in *table 4*. Centered defuzzification method is used to convert the fuzzy output into a single crisp control signal.

Table 4. Rule Table of Fuzzy Logic Controller

e \ Δe	NL	NS	ZE	PS	PL
NL	NL	NL	NL	NL	NL
NS	NS	NS	NS	NS	NS
ZE	ZE	ZE	ZE	ZE	ZE
PS	PS	PS	PS	PS	PS
PL	PL	PL	PL	PL	PL

The servo response of the quadruple tank system with dead time, using a fuzzy logic controller for setpoint changes from 0 to 15 cm and 15 to 20 cm in Tank1, and from 0 to 10 cm and 10 to 5 cm in Tank2, is represented in *figure 5(a)*. The corresponding variation in the controller output is depicted in *figure 5(b)*. From these responses and *table 4*, it is observed that the fuzzy logic controller produces lower ISE and IAE values compared to the PI controller. However, the fuzzy logic controller results in an equilibrium error between the reference input and the actual output. This necessitates the design of subtractive fuzzy PI controllers for a nonlinear quadruple tank system.

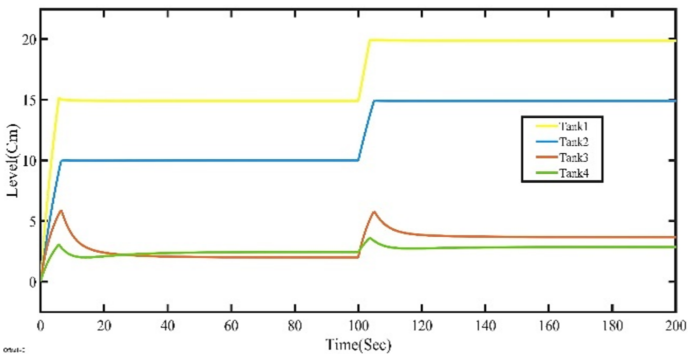

Figure 5(a). Process Output

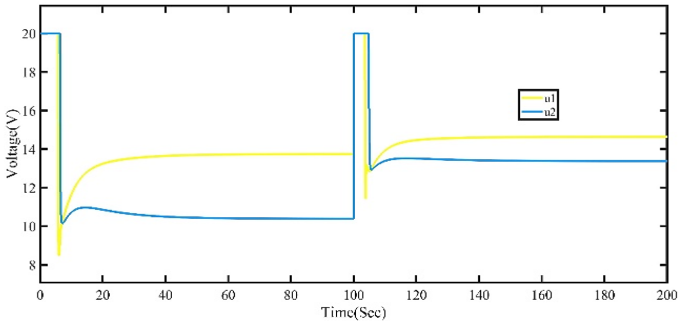

Figure 5(b). Controller Output

Figure 5. Servo response of quadruple tank process with dead time with Fuzzy logic controller

Table 5. Performance Measures of QTSWDT with Fuzzy Logic Controller

Setpoint Change	Tank1		Tank2	
	0 to 15 cm	**15 to 20cm**	**0 to 10 cm**	**10 to 15 cm**
ISE	491.9	47.8	241.3	53.9
IAE	45.55	21.01	31.46	21.01
ITAE	554.7	2011.3	72.8	2573.2
RMSE	0.50	0.012	0.49	0.009
Risetime(sec)	4.46	2.89	5.13	3.99
Settling time (sec)	16.44	13.78	27.26	6.08

6. SUBTRACTIVE FUZZY PI CONTROLLER

Subtractive fuzzy clustering is a powerful and efficient approach employed to automatically create fuzzy rules and membership functions from data for fuzzy inference systems. It combines subtractive clustering with fuzzy logic to model complex systems. Subtractive clustering identifies potential cluster centers based on data density, where each data point's potential is calculated considering its surrounding points within a defined radius. Points with the highest potential are selected as cluster centers, and nearby data points within the radius of influence are suppressed to ensure distinct clusters. This process continues iteratively until all significant clusters are identified. The identified clusters are then used to generate fuzzy rules, with each cluster representing a region of the input-output space. The fuzzy membership functions are derived from the cluster properties, enabling smooth transitions between clusters. A key advantage of subtractive fuzzy clustering is its ability to model non-linear and high-dimensional systems efficiently, with the cluster radius controlling the granularity of the fuzzy model. It does not require prior knowledge of the number of clusters, making it adaptive to various datasets. This technique is widely used in system modeling, control design, and predictive analytics, offering robust performance in handling uncertainties and non-linear relationships.

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A subtractive fuzzy clustering-based PI controller is designed to enhance the control of a quadruple tank system with dead time, a challenging MIMO system characterized by nonlinear dynamics and interactions. Subtractive clustering identifies clusters in the system data, generating fuzzy rules and membership functions to dynamically tune the PI controller's gains, K_i and K_p. This approach improves the controller's ability to manage the system's minimum and non-minimum phase behaviours, coupling effects, and uncertainties. The membership functions and fuzzy rules of the automatically generated subtractive clustering-based Fuzzy PI controller are depicted in *figure 6*.

6.1. Algorithm

The algorithm flow for designing subtractive fuzzy PI controller is as follows.
Step 1: Start
Step 2: Acquire Process Data
Step 3: Preprocess the Data
Step 4: Apply Subtractive Clustering
Step 5: Identify Cluster Centers and Generate Membership Functions
Step 6: Generate Fuzzy Rules Automatically
Step 7: Tune PI Gains Dynamically
Step 8: Defuzzification
Step 9: Apply Control Signals to QTSwDT.
Step 10: Stop

6.2. Implementation in MATLAB/Simulink

1. Develop nonlinear QTSwDT model.
2. Obtain open-loop data.
3. Design PI controller using Z-N tuning.
4. Design Mamdani FLC using Fuzzy Logic Toolbox.
5. Generate SFC-PI controller using genfis2() with optimal controller parameters
6. Export generated FIS to Simulink.
7. Connect fuzzy outputs to adaptive Kc and Ki blocks.
8. Implement PI control equation.
9. Simulate servo and regulatory tests.
10. Evaluate ISE, IAE, ITAE, RMSE, rise time and settling time

The servo response of the quadruple tank process with dead time, controlled by a subtractive Fuzzy PI controller for setpoint changes from 0 to 15 cm and 15 to 20 cm in Tank1, and from 0 to 10 cm and 10 to 15 cm in Tank2, is presented in *figure 7(a)*. *Figure 7(b)* illustrates the corresponding change in the controller output. From these responses and *table 5*, it is observed that the subtractive Fuzzy PI controller produces less overshoot, as well as lower ISE and IAE, compared to the conventional PI and Fuzzy Logic controllers. The parameters used for subtractive clustering are shown in *table 6*.

Table 6. Subtractive Clustering Parameters

S. No.	Name of the Subtractive Clustering Parameter		Obtained Value
1	Clustering radius		0.5
2	Acceptance ratio		0.5
3	Rejection ratio		0.15
4	Number of clusters generated		5
5	Tuning parameters	Kc	4 to 7
		Ki	0.5 to 1.5

A dataset consisting of 2,000 input–output samples was used for fuzzy model identification. The input variables were the control error e and the change in error c_e, while the outputs were the proportional gain Kc and the integral gain *Ki*. Samples ranging from 1,001 to 3,000 were extracted from the simulation data and used to generate the fuzzy inference system through subtractive clustering with a cluster radius of 0.5. The input variables error *e* and change in error c_e, are represented in the input matrix *X*, and the output variables *Kc* and *Ki*, are represented in the output matrix *Y* as shown below.

$$X=\begin{pmatrix} 0.90 & 0.01 \\ 0.81 & 0.01 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 0.55 & 0.01 \end{pmatrix} \qquad Y=\begin{pmatrix} 4.55 & 0.69 \\ 4.48 & 0.66 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 4.33 & 0.61 \end{pmatrix}$$

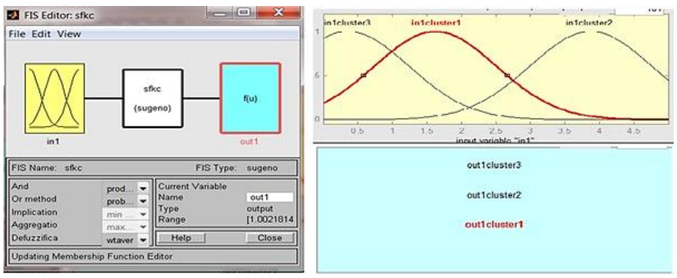

Figure 6. Subtractive Fuzzy clustering automatically generated Input membership functions output

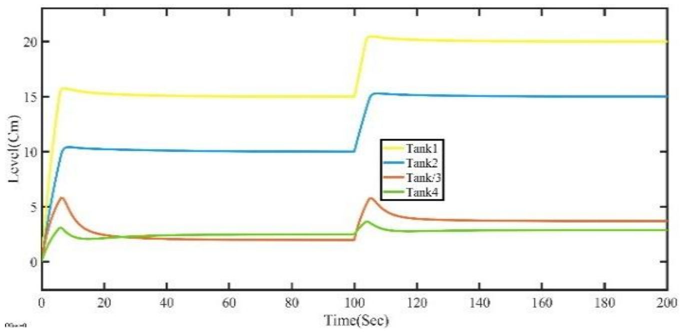

Figure 7(a). Process Output

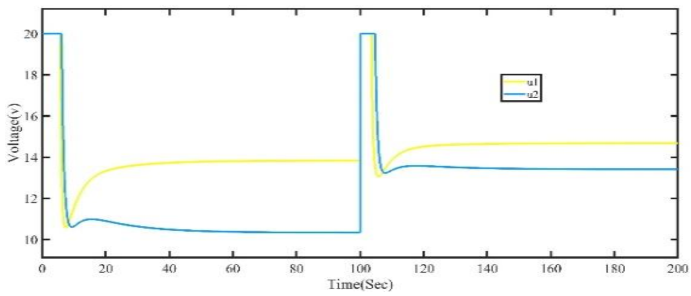

Figure 7(b). Controller Output
Figure 7. Servo response of quadruple tank process with dead time with Subtractive Fuzzy PI controller

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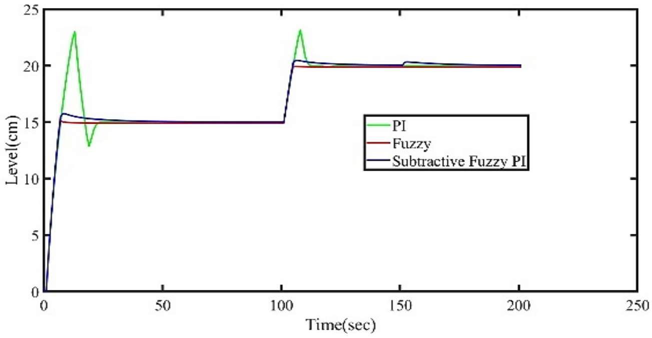

Figure 8(a). Process Output

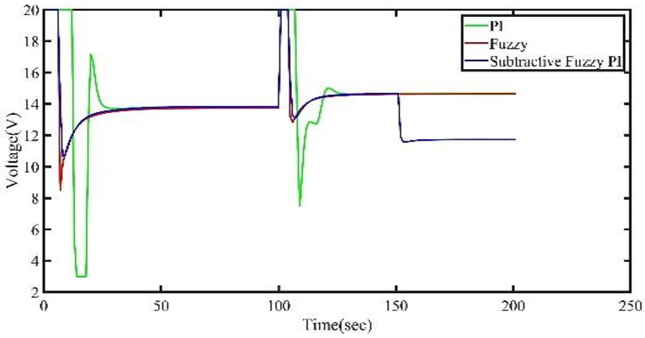

Figure 8(b). Controller Output

Figure 8. Comparison of Servo response of Tank1 with PI, Fuzzy and Subtractive Fuzzy PI controller

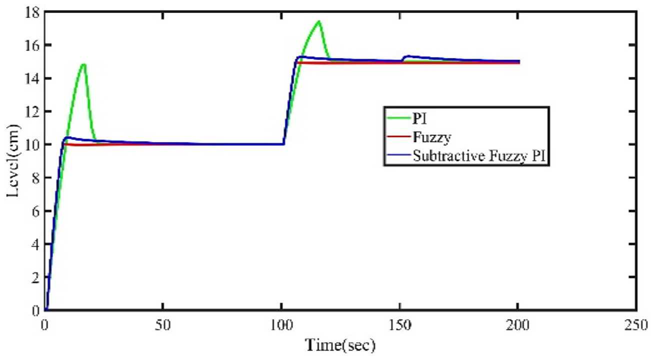

Figure 9(a). Process Output

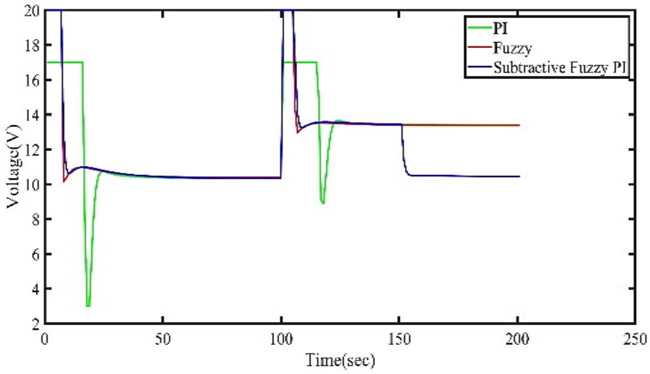

Fig 9(b). Controller output

Figure 9. Comparison of Servo response of Tank2 with PI, Fuzzy and Subtractive Fuzzy PI controller

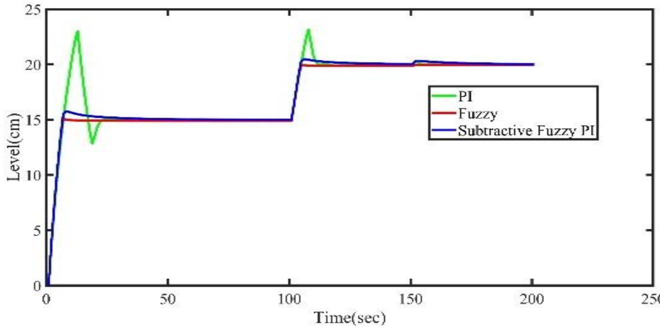

Figure 10(a). Process Output

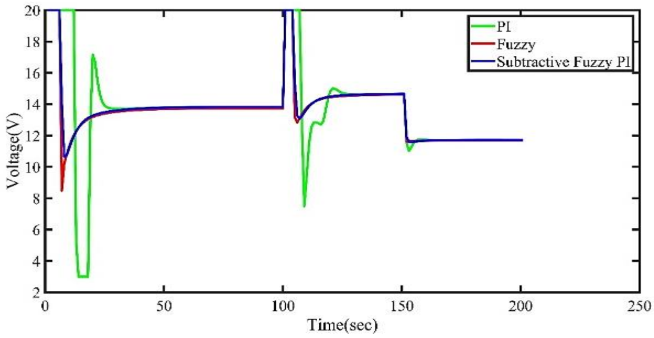

Figure 10(b). Controller Output

Figure 10. Comparison of regulatory response of Tank1 with PI, Fuzzy and Subtractive Fuzzy PI controller

Table 8 compares the performance of the PI, Fuzzy, GWO-ANFIS, and SFC-PI controllers for the quadruple tank system. The conventional PI controller exhibits the poorest performance, with the highest values of ISE (748.7), IAE (88.94), ITAE (666.7), RMSE (2.501), and settling time (23 s). The Fuzzy controller significantly improves the control performance by reducing the tracking error indices and settling time. Further enhancement is achieved by the GWO-ANFIS controller, which utilizes Grey Wolf Optimization to tune the ANFIS structure, resulting in lower error measures and faster dynamic response. Among all controllers, the proposed SFC-PI controller achieves the best overall performance. It yields the lowest ISE (484.5), IAE (43.5), ITAE (316.4), and RMSE (0.49), indicating superior tracking accuracy and disturbance rejection capability.

7. RESULTS AND DISCUSSION

7.1. Simulation Environment and Configuration

Table 8. Performance comparison of controllers for the QTSWDT

Parameter	PI	Fuzzy	GWO based ANFIS	Subtractive Fuzzy PI
ISE	748.7	491.9	488	484.5
IAE	88.94	45.55	44.2	43.5
ITAE	666.7	554.7	420	316.4
RMSE	2.501	0.50	0.495	0.49
Risetime (sec)	4.77	4.46	4.4	4.35
Settling time(sec)	23	16.44	14.5	12.266

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All simulations were performed using MATLAB/Simulink R2022a. The quadruple tank system with dead time was modelled as a continuous-time state-space system and simulated using the variable-step ode45 solver. The simulation duration was set from 0 to 200 seconds. Since a continuous-time model was employed, no discrete sampling time was specified. The system configuration details are as follows:

Processor (CPU): Intel(R) Core (TM) i5-1135G7 @ 2.40 GHz
Installed RAM:8GB
System Type: x64-based PC
OS: Windows 10 Home
System Manufacturer and Model: HP Pavilion laptop 14-ce3xxx

7.2. Comparison of Servo and Regulatory Response Analysis

The comparison of servo responses of the quadruple tank system's Tank 1 level control with PI, Fuzzy, and Subtractive Fuzzy PI controllers for setpoint changes from 0 to 15 cm and from 15 to 20 cm is shown in *figure 8(a)*. The corresponding changes in the controller output are illustrated in *figure 8(b)*. Similarly, the comparison of servo responses for Tank2 level control with PI, Fuzzy, and Subtractive Fuzzy PI controllers for setpoint changes from 0 to 10 cm and from 10 to 15 cm is shown in *figure 9(a)*. The related controller output variations are shown in *figure 9(b)*. The regulatory response of the quadruple tank system was obtained by introducing a step disturbance of magnitude 0.5 at \(t = 100\,s\). The comparison of regulatory responses for Tank1 level control with PI, Fuzzy, and Subtractive Fuzzy PI controllers for setpoint changes from 0 to 15 cm and from 15 to 20 cm is presented in *figure 10(a)*. The corresponding variation in the controller output is shown in *figure 10(b)*. Similarly, the comparison of regulatory responses for Tank2 level control with the same controllers for setpoint changes from 0 to 10 cm and from 10 to 15 cm is shown in *figure 11(a)*. The associated controller output variations are depicted in *figure 11(b)*. *Table 5* shows that the IAE and ISE values for the Subtractive Fuzzy PI controller, for both Tank1 and Tank2, are lower compared to those of the PI and Fuzzy controllers. Based on these responses and the data in *table 5*, it is evident that the Subtractive Fuzzy PI controller produces smoother output and shorter settling times compared to the PI and Fuzzy PI controllers.

Despite the absence of experimental validation or hardware-in-the-loop implementation in this study due to laboratory and hardware constraints, the practical viability of the proposed subtractive fuzzy clustering-based PI controller has been thoroughly evaluated. The controller was developed using a realistic nonlinear four-tank process model that includes dead time, accurately reflecting actual industrial liquid-level systems. The designed controller framework combines traditional PI control with fuzzy clustering methods, making it computationally straightforward and suitable for real-time applications using industrial controllers such as PLCs, DCS, or embedded microcontrollers.

7.3. Stability Analysis

The closed-loop stability of the proposed controller is confirmed *via* eigenvalue analysis of the closed-loop state matrix D - EK. Here D is the system matrix, E is the input matrix, and K is the controller gain matrix. Incorporating the parameter values from *table 1* into *equation (7)* the matrices *D* and *E* are derived as shown below.

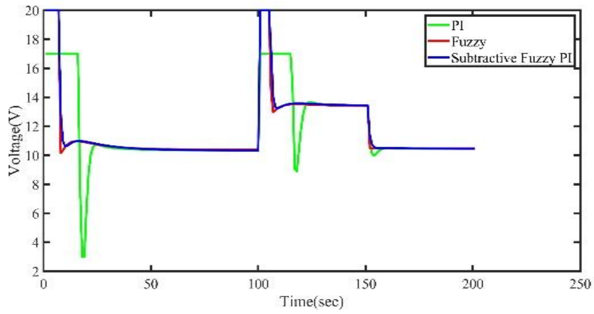

Figure 11(a). Process Output

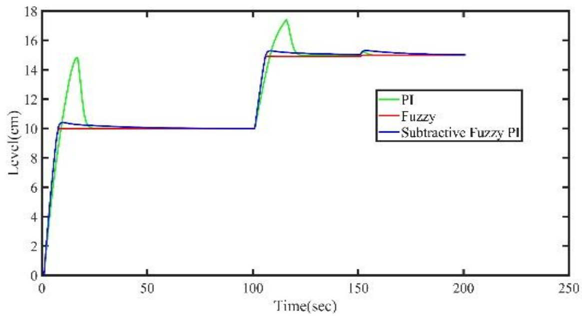

Figure 11(b). Controller Output
Figure 11. Comparison of regulatory response of Tank2 with PI, Fuzzy and Subtractive Fuzzy PI

$$D=\begin{pmatrix} -0.33 & 0 & 0.27 & 0 \\ 0 & -0.20 & 0 & 0.20 \\ 0 & 0 & -0.27 & 0 \\ 0 & 0 & 0 & -0.16 \end{pmatrix}$$

$$E=\begin{pmatrix} 0.20 & 0 \\ 0 & 0.11 \\ 0 & 0.13 \\ 0.06 & 0 \end{pmatrix}$$

$$K=\begin{pmatrix} -5.50 & 110.66 & -94.76 & 63.85 \\ 26.28 & 125.74 & -88.60 & -71.08 \end{pmatrix}$$

The controller gain matrix \(K\) and the eigenvalues of the system matrix in the closed loop were calculated using MATLAB. The obtained eigenvalues are -3, -2, -1, -1 and these values are negative and real. Hence, the system is stable.

7.4. Computational burden and implementation feasibility

The computational burden of the proposed SFC-PI controller is reasonable, since the subtractive clustering and rule-generation tasks are conducted offline during the design phase. As a result, the online execution involves only fuzzy inference and PI calculations, making it suitable for real-time applications. Although the controller requires somewhat more computational resources than a standard PI controller, its demands still fall within the capabilities of contemporary industrial control hardware. Furthermore, the automatic creation of fuzzy rules reduces the need for manual adjustments and enhances the practicality of implementation.

8. CONCLUSION AND FUTURE WORK

This research introduced the development of a Subtractive Fuzzy Clustering-Based PI controller for managing the level control of a quadruple tank system with dead time. The effectiveness of the proposed controller was evaluated against traditional PI and Fuzzy Logic Controllers. The findings show that, although the traditional PI controller is straightforward and easy to use, its efficacy is limited by the nonlinear dynamics, dead time, and interactions within the quadruple tank system. The Fuzzy Logic Controller enhances system efficiency; however, it may still exhibit discrepancies between the reference input and the actual output. Conversely, the SFC-PI controller successfully combines the benefits of PI control with the generation of fuzzy rules based on data, resulting in greater tracking precision, reduced settling times, increased robustness, and improved overall control performance. These features make it a viable option for real-world industrial applications that require managing multivariable processes, including those in the chemical, water treatment, and process sectors.

Although it exhibits enhanced performance, the proposed method has certain limitations, such as dependence on the quality of training data and validation under specific operating conditions. Future research could focus on real-time experimental implementations, examining controller effectiveness under various disturbances and uncertainties. Additionally, incorporating optimization techniques for the automatic adjustment of controller parameters could further improve performance. Furthermore, hybrid intelligent control strategies that combine subtractive fuzzy clustering with advanced methods, such as Model Predictive Control and machine learning techniques, could be explored to enhance control accuracy, flexibility, and robustness in complex nonlinear systems.

Although the proposed controller demonstrated superior performance in the conducted simulations, future work should include repeated simulation and experimental studies, accompanied by statistical analyses—such as mean error, standard deviation, and confidence intervals—to further quantify its reliability and robustness.

Conflicts of Interest: “The authors declare no conflict of interest.”

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