FOREX Publication
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International Journal of
Electrical and Electronics Research (IJEER)
Research Article | Volume 14, Issue 2 | Pages 572-580| e-ISSN: 2347-470X

IOSS Criterion for Lipschitz Nonlinear Fixed Point Discrete -Time Systems Employing External Interference and Saturation Overflow

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ABSTRACT- This paper introduces a novel criterion for assessing the Input-Output-to-State Stability of Lipschitz nonlinear discrete-time systems subject to external interference and saturation arithmetic. By using the Lipschitz condition along with the 'passivity property' of saturation arithmetic and the Lyapunov stability concept, the suggested criterion ensures the suppression of the effects of external interference while guaranteeing asymptotic stability without considering such interference. Two examples are presented to illustrate the effectiveness of the suggested results.

Keywords: Asymptotic Stability, Discrete-Time System, Lipschitz Nonlinear System, Saturation Overflow, Input-Output-To-State Stability (IOSS).

ARTICLE INFORMATION
Author(s): Swagatika Nayak, Smita Rani Parija, Pushpendra Kumar Gupta, and V. Krishna Rao Kandanvli;
Received: 12/07/25; **Accepted**: 15/05/26; **Published**: 30/06/26;
E- ISSN: 2347-470X;
Paper Id: IJEER250129;
Citation: 10.37391/ijeer.140232
Webpage-link:
https://ijeer.forexjournal.co.in/archive/volume-14/ijeer-140232.html

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1. INTRODUCTION

Discrete-time systems (DTSs) are essential in everyday electronics, with widespread applications in the field of telecommunications, video signal enhancement, speech processing, navigation systems, and more [1-4]. Nonlinearities can arise due to quantization and/or overflow when implementing digital hardware/processors that use fixed-point arithmetic [5-7]. Such nonlinearities can cause instability in the designed systems and may also result in self-sustaining oscillations, referred to as limit cycles [8]. Global asymptotic stability (GAS) of the zero solution prevents limit cycles in the implemented DTSs [9,10]. The stability analysis of DTSs employing saturation overflow (SO) has been provided by [9,11,12]. High-order recursive DTSs are generally sensitive to nonlinearities introduced by finite word length effects. When implementing these high-order DTSs in hardware, they must be divided into multiple biquad systems. In such implementations, external interferences (EIs) between biquad systems are unavoidable, which may cause system instability [5,6].

Due to the large number of applications in areas such as nonlinear filtering, state estimation, and telecommunications, Lipschitz nonlinear DTSs can be classified as a significant class of systems. By using Lipschitz continuity, the input-to-state stability (ISS) results for DTSs under SO and EIs have been presented in [12].

The input-output-to-state stability (IOSS) framework is an effective method for ensuring the stability of DTSs [13]. The IOSS approach implies that, regardless of the system's initial state, if the input and observed output remain small, the state will eventually become small as well. Subsequently, in [14], a novel criterion was formulated to evaluate the IOSS of DTS with finite word length nonlinearities and EIs. In [15], a criterion for IOSS to prevent limit cycles in DTS with SO and EIs has been provided. The impact analysis of EI in fixed-point DTS, considering quantization and overflow nonlinearities, is presented in [16]. In [17], the criteria for stability analysis of DTSs subjected to SO and EIs were presented. The augmented Lyapunov function (LF) was employed in [18] to analyze the GAS of DTSs with SO and EIs.

The realistic DTSs often operate with inherent system nonlinearities, overflow nonlinearities and EIs. It is more practical to study the stability of DTSs under these conditions. To the best of our knowledge, no study in the literature has yet considered the stability of such DTSs using the IOSS approach. Inspired by the previous discussion, we address the IOSS of DTSs with SO, Lipschitz nonlinearity and EIs. The combination of SO, inherent system nonlinearities, and EI makes the present system different from the systems considered in [9,12,14-16].

The primary contributions of this work are as follows:

1. A novel IOSS criterion is presented for Lipschitz nonlinear DTSs with SO and EI.
2. In addition, a further stability condition for the DTSs to achieve asymptotic stability (AS), without taking EI into account, is established.
3. The presented criteria are based on LMIs and the feasibility of the criteria can be verified with the help of MATLAB and YALMIP software [19,20].

The subsequent sections of this paper are structured as follows: *Section 1* outlines the considered system, along with key definitions, and lemmas. *Section 2* provides a comprehensive review of the relevant literature. *Section 3* outlines the key findings of the paper. Numerical examples are given in *section 4* to examine the applicability of the proposed results. In *section 5*, a detailed discussion of the obtained results is provided. Lastly, *section 6* presents the conclusions and future work.

Notations: R^n shows the set of $n\times 1$ real vectors; $R□^{n\times m}$ represents $n\times m$real Euclidean spaces along with the standard Euclidean norm $|\cdot|$; $\lambda(C)_{min}$ and $\lambda(C)_{max}$ refer minimum and maximum eigen values of matrix C; $C^T$is transpose of C; C >0 and C <0 are positive definite symmetric and negative definite symmetric matrices, respectively;$sup\{\cdot\}$is least or supremum bound of a set; 0represents null matrix or null vector.

1.1. System Description

The considered nonlinear DTS is represented as follows:

$$\Phi(\Theta+1)=\Lambda\Big(C\Phi(\Theta)+Q\hbar\big(\Phi(\Theta)\big)\Big)+S\zeta(\Theta), \tag{1a}$$

$$\xi(\Theta)=C\Phi(\Theta)+F\zeta(\Theta), \tag{1b}$$

where$\Phi(\Theta)\in R^n$ is the state vector, $\zeta(\Theta)\in R^m$ represents the EI, $\xi(\Theta)$ is the output vector, $\hbar\big(\Phi(\Theta)\big)$ denotes system nonlinearity, $\Lambda(\cdot)\in R^n$represents the SO, $C\in R^{n\times n}$ is coefficient matrix, $Q\in R^{n\times n}$matrix associated with system nonlinearity, $S\in R^{n\times m}$and $F\in R^{n\times m}$ are known constant matrices.

The system nonlinearity $\hbar\big(\Phi(\Theta)\big)$ with Lipschitz constant L is said to be Lipschitz continuous for each vector of $\overline{\Phi_1}(\Theta),\overline{\Phi_2}(\Theta)\in R^n$ and time instant Θ, $\hbar\big(\Phi(\Theta)\big)$ satisfies the following condition

$$\left\|\hbar\left(\overline{\Phi_1}(\Theta)\right)-\hbar\left(\overline{\Phi_2}(\Theta)\right)\right\|\le L\left\|\overline{\Phi_1}(\Theta)-\overline{\Phi_2}(\Theta)\right\|. \tag{2}$$

The SO under consideration is presented by

$$\Lambda_Z\big(\Upsilon_Z(\Theta)\big)=\begin{cases}1 if\,\Upsilon_Z(\Theta)>1,\\ \Upsilon_Z(\Theta)if-1\le\Upsilon_Z(\Theta)\le 1, Z=1,2,\ldots,n.\\ -1if\,\Upsilon_Z(\Theta)<-1,\end{cases}$$

(3)

1.2. Preliminaries

This paper utilizes the following definitions to derive the main results:

Definition1: [15,21] A function $\chi\colon R^+\to R^+$is defined as class$K$when it is continuous, exhibits strict increment and $\chi(0)=0$. A function $\psi\colon R^+\to R^+$is defined as class $K_\infty$when $\psi\in K$and$\psi(\omega)\to\infty$as $\omega\to\infty$.

Definition2: [12,21] A function $\varphi\colon R^+\to R^+$is defined as class $L$when it is continuous, exhibits strict decrement and $\varphi(\omega)=0$ as $\omega\to\infty$.

Definition3: [15,21] A function $\gamma\colon R^+\times R^+\to R^+$ is defined as class KL if for each fixed $\eta\ge 0,\gamma(\cdot,\eta)\in K$ and for each fixed $\omega\ge 0,\gamma(\omega,\cdot)\in L$.

Definition4: [15,21] The DTS (1) and (2) is said to be IOSS if there exist K function $\psi_1(\omega)$, $\psi_2(\omega)$ and a KL function $\gamma(\omega,\eta)$, such that for each input ζ_Θ, each output $\xi_\Theta$and each initial state Φ_0, we have

$$\|\boldsymbol{\Phi}_\Theta\|\le\max\left\{\gamma\left(\|\boldsymbol{\Phi}_0\|,\eta\right),\ \psi_1\left(\sup_{0\le i\le\Theta}\|\zeta_i\|\right),\ \psi_2\left(\sup_{0\le i\le\Theta}\|\xi\|_i\right)\right\}\quad\forall\Theta\ge 0. \tag{4}$$

Lemma 1: [15,21] A function $\Im_\Theta$ is termed an IOSS- LF for the given DTS (1) if there are K_∞ functions φ_1, φ_2,φ_3 and K functions φ_4,φ_5, satisfying the conditions:

$$\varphi_1(\|\Phi_\Theta\|)\le\Im_\Theta\le\varphi_2(\|\Phi_\Theta\|), \tag{5}$$

for any$\Phi_\Theta\in R^n$and

$$\Im_{\Theta+1}-\Im_\Theta\le-\varphi_3(\|\Phi_\Theta\|)+\varphi_4(\|\zeta_\Theta\|)+\varphi_5(\|\xi_\Theta\|), \tag{6}$$

for any $\boldsymbol{\Phi}_\Theta\in\boldsymbol{R}^n,\zeta_\Theta\in R^m$and $\xi_\Theta\in R^p$. Then, the DTS (1) is IOSS if and only if it possesses an IOSS- LF.

Lemma 2: [22] Let $n\times n$ positive definite symmetric matrices $\lambdabar_1=[\lambdabar_{ij}]$and $\lambdabar_2=[\bar{\lambdabar}□_{ij}]$which satisfy,

$$\begin{aligned}&\big[C\Phi(\Theta)+Q\hbar\big(\Phi(\Theta)\big)\big]^T\lambdabar_1\big[C\Phi(\Theta)+Q\hbar\big(\Phi(\Theta)\big)\big]\\&-\Big[\Lambda^T\Big(C\Phi(\Theta)+Q\hbar\big(\Phi(\Theta)\big)\Big)\\&\times\lambdabar_1\Lambda\Big(C\Phi(\Theta)+Q\hbar\big(\Phi(\Theta)\big)\Big)\Big]\ge 0,\end{aligned} \tag{7}$$

if and only if,

$$\lambdabar_{ii}\ge\textstyle\sum_{j=1j\ne i}^n\left|\lambdabar_{ij}\right|, i=1,2,\ldots,n, \tag{7a}$$

and

$$\begin{aligned}&[C\Phi(\Theta)]^T\lambdabar_2[C\Phi(\Theta)]-\Big[\Lambda^T\Big(C\Phi(\Theta)+Q\hbar\big(\Phi(\Theta)\big)\Big)\\&\times\lambdabar_2\Lambda\Big(C\Phi(\Theta)+Q\hbar\big(\Phi(\Theta)\big)\Big)\Big]\ge 0,\end{aligned} \tag{8}$$

if and only if

$$\bar{\lambdabar}_{ii}\ge\textstyle\sum_{j=1j\ne i}^n\left|\bar{\lambdabar}_{ij}\right|, i=1,2,\ldots,n. \tag{8a}$$

2. LITERATURE REVIEW

In recent decades, DTSs have gained significant attention across various research domains. In [23], the stability of 1-D dynamical systems subjected to nonlinear disturbances has been investigated. In DTSs, arithmetic operations such as addition and multiplication can increase the required word length for storing intermediate results [24]. To control this overflow, word length reduction techniques are applied [8]. However, these techniques can introduce nonlinear effects, including overflow and quantization errors [2,16,25].

Furthermore, overflow induced nonlinearities can severely distort processed signals and may produce high-amplitude limit-cycle oscillations [9]. These oscillatory behaviors are undesirable, as they can eventually reduce the stability of the system [17]. In DTSs, common overflow handling mechanisms include zeroing, triangular, saturation, and two's complement methods are given in [26, 27]. Among these, SO provides a comparatively larger stability region, making it more favorable and extensively investigated in the literature [10,11,14,15,22,23,28,29]. In [7,31,34,36], limit cycle-free realizations were investigated for interfered DTSs with time-varying delays and SO, ensuring oscillation-free performance. In [32], overflow oscillation-free realization for state-delayed systems with SO arithmetic was addressed. Additionally, [33] presents improved stability and passivity conditions for delayed DTSs with SO nonlinearities and EIs using advanced Lyapunov techniques.

When high-order filters are realized through the interconnection of multiple first and/or second-order DTSs, the occurrence of EI is inevitable in such implementations [15,23,25]. Consequently, the analysis of DTSs in the presence of EIs has become a prominent research topic and continues to receive sustained attention in the literature [11,12,16,17,37]. Passivity, regarded as a special case of dissipative, characterizes the energy exchange properties of a system. According to passivity theory, a passive system ensures the stability of its internal states. This property has been extensively applied in controller design, stability analysis, and state estimation of dynamical systems. Furthermore, a passivity-based condition for DTSs with EI operating under SO arithmetic was proposed in [29].

Recent studies have extensively addressed stability analysis of DTSs under delays, uncertainties, and nonlinear effects. In [38], the authors consider a class of 2-D discrete systems described by the Roesser model, incorporating parametric uncertainties and bounded input SO nonlinearities. To analyze system stability, they employ a Lyapunov–Krasovskii functional approach and derive sufficient conditions in terms of Linear Matrix Inequalities (LMIs). These conditions ensure AS of the DTS despite the combined effects of delays, uncertainties, and SO constraints. In [39], an ISS-based approach was developed to ensure stability under EI and state delays. The study employs the concept of ISS combined with LF-based techniques to derive novel stability criteria. Further, [40] proposes novel delay-dependent stability conditions formulated as LMIs using a LF-based framework. The approach effectively addresses time-varying delays and nonlinear SO effects while reducing conservatism. Moreover, in [41], a passivity-based framework is introduced to guarantee exponential stability under overflow nonlinearities. By employing LF and passivity concepts, the authors derive stability conditions that ensure robustness against finite word-length effects, offering a less conservative and computationally efficient approach.

Existing studies have made significant progress in the stability analysis of DTSs by addressing challenges such as time delays, uncertainties, EIs, and nonlinear effects including SO. Approaches based on LMIs, ISS, and passivity theory have provided effective and less conservative stability conditions, with some works also demonstrating practical implementation through hardware validation. Moreover, efforts toward eliminating limit cycles and overflow oscillations have enhanced system reliability. However, a comprehensive framework that simultaneously incorporates EI, delays, and nonlinear constraints in a unified and efficient manner is still lacking, motivating further research in this area.

To the best of the authors' knowledge, no existing study has addressed the IOSS of Lipschitz nonlinear DTSs under EIs and SO arithmetic using the Lipschitz condition in conjunction with passivity properties. These findings encouraged us to pursue this work.

3. METHODS

This section represents a new IOSS criterion for the interfered Lipschitz nonlinear DTS. The proposed criterion is presented as follows:

Theorem 1: The DTS described by *eq. (1)* is IOSS if there exist $H = H^T > 0, \aleph = \aleph^T > 0, \mathfrak{R} = \mathfrak{R}^T > 0$ and $\Xi = \Xi^T > 0$ such that

$$\mathfrak{W} = \begin{bmatrix} -H + C^T\lambdabar_2 C + L^2\lambdabar_3 + C^T\lambdabar_1 C + \mathfrak{R} & 0 & C^T\lambdabar_1 Q & C^T N \\ * & S^T HS - \aleph & 0 & S^T H + F^T N \\ * & * & -\lambdabar_3 + Q^T\lambdabar_1 Q & 0 \\ * & * & * & H - (N + N^T) - \lambdabar_2 - \lambdabar_1 - \Xi \end{bmatrix} < 0,$$

(9)

where $N \in R^{n\times n}$ is a row diagonally dominant (RDD) matrix having positive diagonal elements, $\lambda_1 = {\lambda_1}^T > 0$, $\lambda_2 = {\lambda_2}^T > 0$ and $\lambdabar_3 = {\lambdabar_3}^T > 0$are diagonally dominant matrices.

Proof: Consider a LF as follows:

$$\mathfrak{I}(\Phi(\Theta)) = \Phi^T(\Theta)H\Phi(\Theta). \tag{10}$$

The function $\mathfrak{I}(\Phi(\Theta))$satisfies the Rayleigh inequality condition as stated in [42]

$$\lambda\|\Phi(\Theta)\|^2\|\Phi(\Theta)\|^2{}_{max\,min} \tag{11}$$

where $H$is positive definite symmetric matrix. Then finding $\Delta\mathfrak{I}(\Phi(\Theta))$ by defining $\Delta\mathfrak{I}\big(\Phi(\Theta)\big) = \mathfrak{I}\big(\Phi(\Theta+1)\big) - \mathfrak{I}\big(\Phi(\Theta)\big)$ along the trajectories of *eq. (1)*, we have

$$\begin{aligned}\Delta\mathfrak{I}\big(\Phi(\Theta)\big) &= \mathfrak{I}\big(\Phi(\Theta+1)\big) - \mathfrak{I}\big(\Phi(\Theta)\big)\\ &= \Phi^T(\Theta+1)H\Phi(\Theta+1) - \Phi^T(\Theta)H\Phi(\Theta)\\ &= \Big[\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big) + S\zeta(\Theta)\Big]^T\\ &\quad\times H\Big[\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &\quad + S\zeta(\Theta)\Big] - \Phi^T(\Theta)H\Phi(\Theta).\end{aligned}$$

$$\begin{aligned}\Delta\mathfrak{I}\big(\Phi(\Theta)\big) &= [\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &\quad\times H\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)]\\ &\quad+ \Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)HS\zeta(\Theta)\\ &\quad+ \zeta^T(\Theta)S^TH\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &\quad+ \zeta^T(\Theta)S^THS\zeta(\Theta) - \Phi^T(\Theta)H\Phi(\Theta)\\ &\quad+ \beta_1 + \beta_2 + \beta_3 + \beta_4.\end{aligned} \tag{12}$$

For the SO $\xi(\Theta)$ given by *eq. (3)*, the following property holds true [28];

$$\begin{aligned}\beta_1 = {}& \xi^T(\Theta)N\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &+\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)N^T\xi(\Theta)\\ &-\Big[\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &\times(N+N^T)\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\Big].\end{aligned} \tag{13}$$

By considering *eq. (3)* and noting that N is a RDD matrix having positive diagonal elements, the quantity β_1 is nonnegative.

In view of [22], β_2 and β_3 can be expressed as

$$\begin{aligned}\beta_2 = {}& \big[C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\big]^T \times \lambdabar_1\big[C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\big]\\ &-\Big[\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &\times \lambdabar_1\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\Big] \geq 0,\end{aligned} \tag{14}$$

$$\begin{aligned}\beta_3 = {}& [C\Phi(\Theta)]^T\lambdabar_2[C\Phi(\Theta)]\\ &-\Big[\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &\times \lambdabar_2\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\Big] \geq 0,\end{aligned} \tag{15}$$

where $\lambdabar_1$is given in (7a) and $\lambdabar_2$ is shown in (8a).

Following [45], it can be shown that $\hbar\big(\Phi(\Theta)\big)$ characterized by *eq. (2)* satisfies

$$\beta_4 = L^2\Phi^T(\Theta)\lambdabar_3\Phi(\Theta) - \hbar^T\big(\Phi(\Theta)\big)\lambdabar_3\hbar\big(\Phi(\Theta)\big) \geq 0. \tag{16}$$

From *eq. (12)-(16)*, it is easy to obtain that

$$\begin{aligned}\Delta\mathfrak{I}(\Phi(\Theta)) \leq {}& \Big[\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &\times H\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\Big]\\ &+\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)HS\zeta(\Theta)\\ &+\zeta^T(\Theta)S^TH\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &+\zeta^T(\Theta)S^THS\zeta(\Theta) - \Phi^T(\Theta)H\Phi(\Theta)\\ &+\xi^T(\Theta)N\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &+\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)N^T\xi(\Theta)\\ &-\Big[\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &\times(N+N^T)\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\Big]\\ &+\big[C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\big]^T\\ &\times\lambdabar_1\big[C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\big]\\ &-\Big[\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &\times\lambdabar_1\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\Big]\\ &+[C\Phi(\Theta)]^T\lambdabar_2[C\Phi(\Theta)]\\ &-\Big[\Lambda^T\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\\ &\times\lambdabar_2\Lambda\Big(C\Phi(\Theta) + Q\hbar\big(\Phi(\Theta)\big)\Big)\Big]\\ &+L^2\Phi^T(\Theta)\lambdabar_3\Phi(\Theta)\\ &-\hbar^T\big(\Phi(\Theta)\big)\lambdabar_3\hbar\big(\Phi(\Theta)\big)\\ &+\Phi^T(\Theta)\mathfrak{R}\Phi(\Theta) - \Phi^T(\Theta)\mathfrak{R}\Phi(\Theta)\\ &+\zeta^T(\Theta)\aleph\zeta(\Theta) - \zeta^T(\Theta)\aleph\zeta(\Theta)\\ &+\xi^T(\Theta)\Xi\xi(\Theta) - \xi^T(\Theta)\Xi\xi(\Theta)\\ &-\beta_1 - \beta_2 - \beta_3 - \beta_4.\end{aligned} \tag{17}$$

$$\Delta\mathfrak{I}\big(\Phi(\Theta)\big) \leq \mathfrak{H}^T(\Theta)\mathfrak{W}\mathfrak{H}(\Theta) + \Phi^T(\Theta)\mathfrak{R}\Phi(\Theta)$$

$$-\zeta^T(\Theta)\aleph\zeta(\Theta) - \xi^T(\Theta)\mathit{\Xi}\xi(\Theta),$$

(18)

Where,

$$\mathfrak{H}^T(\Theta) = [\Phi^T(\Theta) \quad \zeta^T(\Theta)$$

$$\hbar^T\left(\Phi(\Theta)\right) \quad \Lambda^T\left(C\Phi(\Theta) + Q\hbar\left(\Phi(\Theta)\right)\right)]. \tag{19}$$

If $\mathfrak{W} < 0$, we have

$$\Delta\mathfrak{I}\left(\Phi(\Theta)\right) < \Phi^T(\Theta)\mathfrak{R}\Phi(\Theta) - \zeta^T(\Theta)\aleph\zeta(\Theta)$$

$$-\xi^T(\Theta)\mathit{\Xi}\xi(\Theta)$$

$$\leq \lambda(\mathfrak{R})\|\Phi(\Theta)\|^2(\aleph)\|\zeta(\Theta)\|^2{}_{max\,min}$$

$$-\lambda(\mathit{\Xi})\|\xi(\Theta)\|^2{}_{max} \tag{20}$$

Let us define $\varphi_1(\Theta), \varphi_2(\Theta), \varphi_3(\Theta), \varphi_4(\Theta)$and $\varphi_5(\Theta)$, as

$$\varphi_1(\Theta) \triangleq \lambda(H)^2{}_{min} \tag{21a}$$

$$\varphi_2(\Theta) \triangleq \lambda(H)^2{}_{max} \tag{21b}$$

$$\varphi_3(\Theta) \triangleq \lambda(\mathfrak{R})^2{}_{min} \tag{21c}$$

$$\varphi_4(\Theta) \triangleq \lambda(\aleph)^2{}_{max} \tag{21d}$$

$$\varphi_5(\Theta) \triangleq \lambda(\mathit{\Xi})^2{}_{max} \tag{21e}$$

where $\varphi_1(\Theta), \varphi_2(\Theta), \varphi_3(\Theta), \varphi_4(\Theta)$ and $\varphi_5(\Theta)$, are K_∞ functions. Using *eq. (11)* and *(20)*, one can get

$$\varphi_1(\|\Phi(\Theta)\|) \leq \mathfrak{I}(\Phi(\Theta)) \leq \varphi_2(\|\Phi(\Theta)\|), \tag{22}$$

$$\mathfrak{I}(\Phi(\Theta+1)) - \mathfrak{I}(\Phi(\Theta)) \leq \varphi_3(\|\Phi(\Theta)\|) - \varphi_4(\|\zeta(\Theta)\|)$$

$$-\varphi_5(\|\xi(\Theta)\|). \tag{23}$$

In the view of Lemma 1, $\mathfrak{I}\left(\Phi(\Theta)\right)$is an IOSS LF for DTS (1) under the condition (9). Consequently, the DTS (1) is IOSS if (9) is satisfied. Now, Theorem 1 is successfully proven.

Remark 1: To ensure the stability of DTS (1) using the IOSS approach described in Theorem 1, it is necessary to solve (9) for $H > 0, \aleph > 0$ and $\mathfrak{R} > 0$by using MATLAB software [19,20]. Also check whether there exist a solution for N,λ_1, λ_2, λ_3 and $\mathit{\Xi}$. Consequently, this approach basically includes a frequent searching of $H > 0, \aleph > 0$ and $\mathfrak{R} > 0$ that satisfy (9) until a feasible solution for N, λ_1, λ_2, λ_3 and $\mathit{\Xi}$is identified. If (9) yields a feasible solution of DTS (1), then the considered system (1) attains IOSS. Otherwise, a definitive conclusion cannot be made regarding the IOSS stability of the considered system.

Remark 2: Theorem 1 utilizing a quadratic IOSS LF (10), along with condition (14), (15) and (16) with Lipschitz nonlinearities and the passivity characteristic (13) concerning to SO arithmetic. The use of diagonally dominant positive-definite symmetric matrices $\lambda_1, \lambda_2, \lambda_3$ (see (14), (15) and (16), respectively) and a RDD matrix N with positive diagonal elements {*see (13)*} are usually advantageous for getting better IOSS results.

Next, consider the case where the DTS (1) is free from EIs:

$$\Phi(\Theta+1) = \Lambda\left(C\Phi(\Theta) + Q\hbar\left(\Phi(\Theta)\right)\right), \tag{24a}$$

$$\xi(\Theta) = C\Phi(\Theta). \tag{24b}$$

Corollary 1: The DTS represented by (24) is asymptotically stable if there exist $H = H^T > 0, \mathfrak{R} = \mathfrak{R}^T > 0$and $\mathit{\Xi} = \mathit{\Xi}^T > 0$ satisfying

$$\begin{bmatrix} -H + C^T\lambda_2 C + L^2\lambda_3 + C^T\lambda_1 C + \mathfrak{R} & C^T\lambda_1 Q & C^T N \\ * & -\lambda_3 + Q^T\lambda_1 Q & * \\ * & 0 & H - (N + N^T) - \lambda_2 - \mathit{\Xi} \end{bmatrix} < 0, \tag{25}$$

where $N \in R^{n\times n}$ is a RDD matrix with positive diagonal elements, $\lambda_1 = \lambda_1{}^T > 0$, $\lambda_2 = \lambda_2{}^T > 0$ and $\lambda_3 = \lambda_3{}^T > 0$are diagonally dominant matrices.

Proof: Theorem 1 presents an IOSS criterion for the DTS (1) with EI. Now, it remains to show the AS of the DTS (1) with $\zeta(\Theta) = 0$.

When $\zeta(\Theta) = 0$, (20) yields

$$\Delta\mathfrak{I}\left(\Phi(\Theta)\right) < \Phi^T(\Theta)\mathfrak{R}\Phi(\Theta) - \xi^T(\Theta)\mathit{\Xi}\xi(\Theta)$$

$$= \Phi^T(\Theta)[\mathfrak{R} - C^T\mathit{\Xi}C]\Phi(\Theta)$$

$$< 0. \tag{26}$$

This guarantee

$$\lim_{\Theta\to\infty}\Phi(\Theta) = 0, \tag{27}$$

by using Lyapunov theory for stability. Now Corollary 1's proof is completed.

Remark 3: According to [14,15], the IOSS of a system suggests that it remains asymptotically stable when EIs are absent. Consequently, our focus is on analyzing the AS of the DTS (1) with $\zeta(\Theta) = 0$.

Remark 4*:* The MATLAB, in conjunction with YALMIP [19,20], can efficiently validate the feasibility of conditions (9) and (25).

Remark 5: The IOSS technique provides a comprehensive way for tackling the stability difficulties in dynamical systems [21]. It guarantees that the state of the DTS remains small regardless of the initial state, even when both the input and observed output have low magnitudes. As a result, using the IOSS technique, one can evaluate the stability of DTS based on their input-output characteristics.

Remark 6: The system considered in [10,18], having only finite word length nonlinearities (no EI) and in [14,15], employing finite word length nonlinearity and EI. In [14,15], a criterion is presented for IOSS of DTS with SO and EI without considering Lipschitz nonlinearity. Whereas the criterion presented in [12] employs ISS of the DTS with Lipschitz nonlinearity. As per the knowledge of the authors, a unique system is considered, see *eqs. (1)-(3)* in this paper. Also, a criterion for IOSS (see Theorem 1) of Lipschitz nonlinear DTS with SO and EI is proposed.

Figure 1 illustrates the flowchart of the proposed method for a given DTS. The flowchart takes the system parameters(C, S, Q, F, L) of the considered DTS, as described in *equations (1)-(3)*, as input. Subsequently, the validity of the IOSS conditions presented in Theorem 1 is verified using MATLAB together with YALMIP 3.0 [19,20]. If Theorem 1 leads a feasible solution for the considered system, then the system is concluded to be IOSS. However, if no feasible solution is obtained, no conclusion regarding the IOSS of the system can be established.

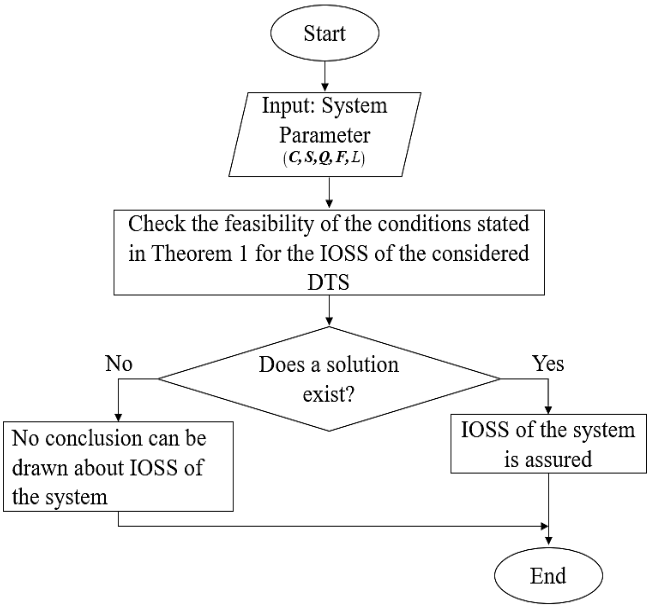


Figure 1. Flow chart for the proposed method

4. RESULTS

To demonstrate the anticipated result, two examples are provided.

Example 1: Consider DTS (1) with

$$C = \begin{bmatrix} 0.1 & -0.35 \\ -0.1 & 0.1 \end{bmatrix}, S = \begin{bmatrix} 0.2 & 0.1 \\ -0.1 & -0.1 \end{bmatrix},$$

$$F = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}. \tag{28}$$

Let EIs as:

$$\zeta(\Theta) = \begin{cases} [0.2n_1(\Theta) \quad -0.3n_2(\Theta)]^T, 0 \le \Theta \le 50, \\ [0 \quad 0]^T, otherwise, \end{cases} \tag{29}$$

Assume that $n_1(\Theta)$ and$n_2(\Theta)$ are random sequences characterized by zero mean, white Gaussian properties and a variance of 1. Consider the nonlinearity of the DTS as

$$\hbar(\Phi(\Theta)) = \begin{bmatrix} tanh(\Phi_1(\Theta)) \\ tanh(\Phi_2(\Theta)) \end{bmatrix}, \tag{30}$$

along with Lipschitz constant $L = 1$.

Utilizing the comprehensive MATLAB software in combination with YALMIP [19,20] to satisfy the condition (9) outlined in Theorem 1, we got the following feasible solutions:

$$H = \begin{bmatrix} 129.4328 & 1.3111 \\ 1.3111 & 145.8792 \end{bmatrix}, \aleph = \begin{bmatrix} 49.2606 & 4.5638 \\ 4.5638 & 44.8592 \end{bmatrix},$$

$$\Re = \begin{bmatrix} 31.1988 & 2.5777 \\ 2.5777 & 32.0331 \end{bmatrix}, N = \begin{bmatrix} 35.7844 & -1.4553 \\ -1.6610 & 38.4018 \end{bmatrix},$$

$$ƛ_1 = \begin{bmatrix} 62.0082 & 1.2493 \\ 1.2493 & 65.1642 \end{bmatrix}, ƛ_2 = \begin{bmatrix} 64.6676 & -1.9685 \\ -1.9685 & 64.9486 \end{bmatrix},$$

$$ƛ_3 = \begin{bmatrix} 64.0434 & 2.8486 \\ 2.8486 & 62.1400 \end{bmatrix}, \Xi = \begin{bmatrix} 27.0877 & -1.0633 \\ -1.0633 & 26.8958 \end{bmatrix} \tag{31}$$

Example 2: Consider the DTS (24) with

$$C = \begin{bmatrix} 0.1 & -0.35 \\ -0.1 & 0.1 \end{bmatrix}, Q = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}. \tag{32}$$

The inherent system nonlinearity of the DTS is assumed as

$$\hbar(\Phi(\Theta)) = \begin{bmatrix} tanh(\Phi_1(\Theta)) \\ tanh(\Phi_2(\Theta)) \end{bmatrix}, \tag{33}$$

along with Lipschitz constant $L = 1$.

By using MATLAB in conjunction with YALMIP [19,20] to address the condition (25) in Corollary 1, we found the feasible solutions as follows:

$$H = \begin{bmatrix} 19.5224 & 0.0209 \\ 0.0209 & 20.3092 \end{bmatrix}, \aleph = \begin{bmatrix} 10.3056 & 0 \\ 0 & 10.3056 \end{bmatrix},$$

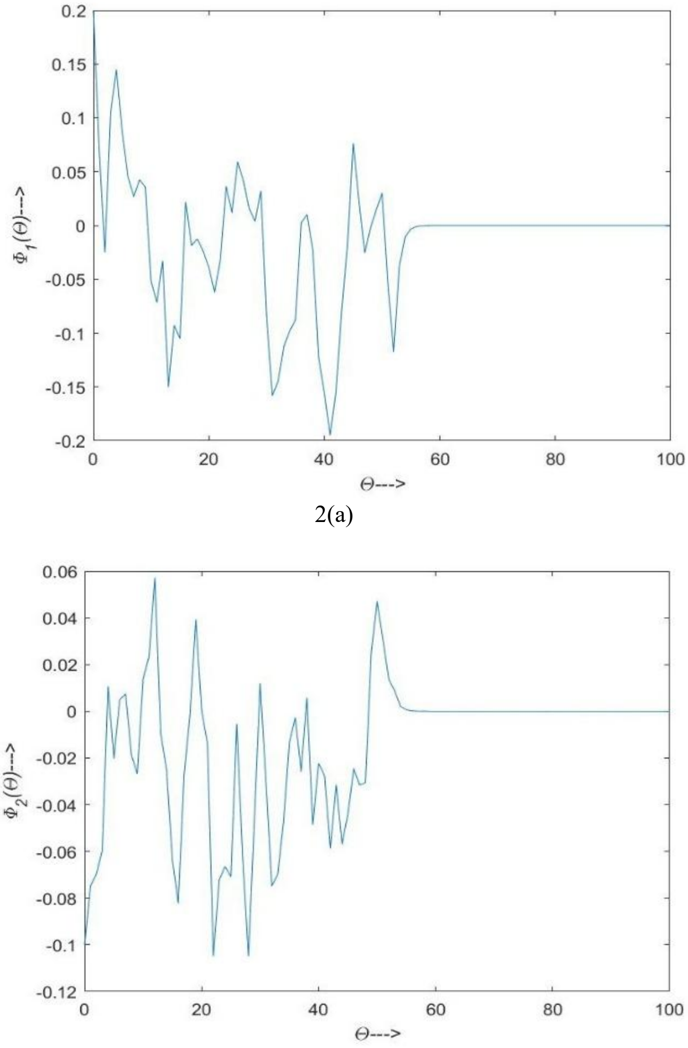
$$\Re = \begin{bmatrix} 4.4913 & 0.2347 \\ 0.2347 & 4.1453 \end{bmatrix}, ƛ_1 = \begin{bmatrix} 6.9330 & 0.7046 \\ 0.7046 & 7.7777 \end{bmatrix},$$

$$\hat{\lambda}_2 = \begin{bmatrix} 10.2865 & 0.1148 \\ 0.1148 & 10.5505 \end{bmatrix}, \lambda_3 = \begin{bmatrix} 10.2008 & 0.3005 \\ 0.3005 & 9.7834 \end{bmatrix},$$

$$N = \begin{bmatrix} 4.5540 & -0.0996 \\ -0.1877 & 4.8337 \end{bmatrix}, \Xi = \begin{bmatrix} 6.9032 & -0.2556 \\ -0.2556 & 6.4624 \end{bmatrix} \quad (34)$$

Thus, by Corollary 1, the system under consideration is asymptotically stable when $\zeta(\Theta) = 0$.

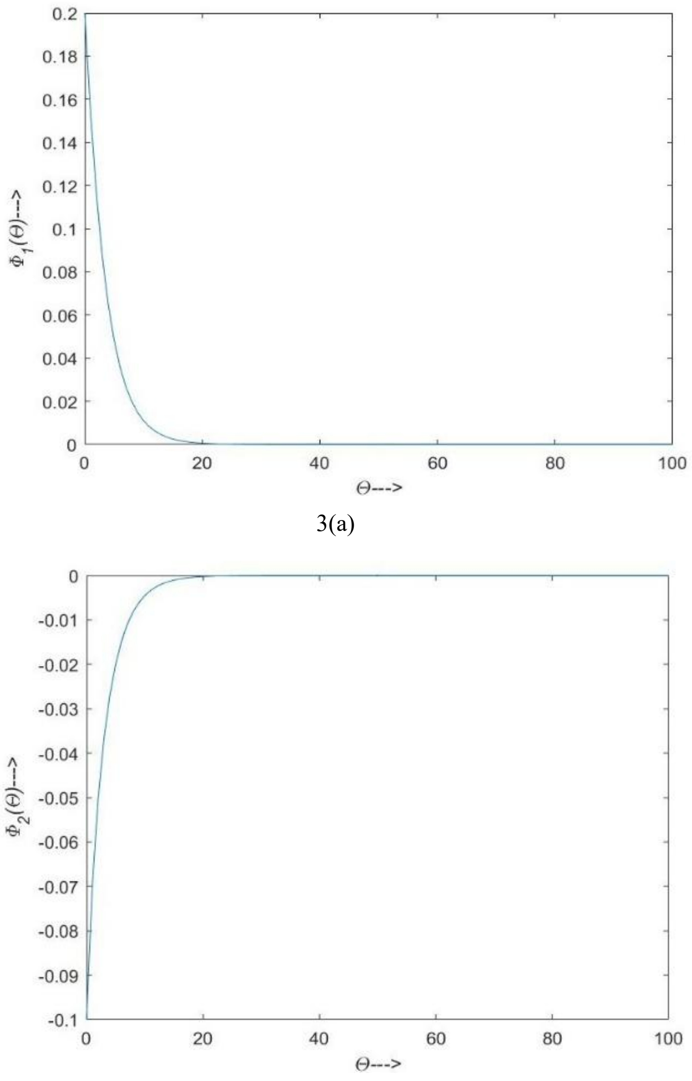
Simulation results are provided using examples for validating Theorem 1 as well as Corollary 1. The initial condition is assumed as $\Phi(0) = [0.2 - 0.1]^T$ and EIs are outlined in *eq. (29)* for the designated interval $0 \leq \Theta \leq 50$. The state trajectories of systems in Examples 1 and 2 are depicted in *figures (2)* and *(3)*, respectively. *Figure 2* shows that the state trajectories of DTS (1) stays within bounded limit. From *figure 3*, it can be seen that the state trajectory of system considered in *example 2* converges to zero. Thus, from the simulation results, IOSS is assured for the considered DTS when $\zeta(\Theta) \neq 0$ and AS is guaranteed when $\zeta(\Theta) = 0$.



2(a)

2(b)

Figure 2. State trajectories of DTS in *example 1*



3(a)

3(b)

Figure 3. State trajectories of DTS in *example 2*

It is worth highlighting that the cases mentioned above go beyond the limitations of the overflow stability criteria presented in [2,16,23,35].

5. DISCUSSION

A sufficient condition for the absence of limit cycles in interfered fixed-point state-space DTSs with SO arithmetic and Lipschitz nonlinearity is derived using the IOSS approach (Theorem 1). Additionally, a criterion (Corollary 1) is proposed to ensure AS in the absence of EI. The proposed result improves upon existing results in [12,14,15].

The validity of the conditions presented in Theorem 1 and Corollary 1 can be readily verified using MATLAB [19] in conjunction with YALMIP 3.0 [20].

While the proposed criterion is helpful for stability analysis, it has certain limitations. Since the approach relies on Lipschitz condition it may only satisfy local Lipschitz continuity.

Extending the approach with global Lipschitz nonlinearities could broaden its applicability. Additionally, the method may not apply all types of nonlinearities.

6. CONCLUSIONS AND FUTURE WORK

A new LMI-based stability criterion for Lipschitz nonlinear DTSs with SO and EI, based on the IOSS approach, has been established in this work. Furthermore, Corollary 1 provides a criterion for the AS of Lipschitz nonlinear DTSs in the presence of SO. The proposed conditions are formulated in terms of LMIs. Also, numerical examples are presented to demonstrate the applicability and effectiveness of the presented results.

Future research may focus on extending these concepts to investigate stability issues in 2-D discrete systems with EI, as well as nonlinear phenomena such as overflow oscillations and quantization effects. The proposed methodology can also be further used for more complex nonlinear systems involving uncertainties, time delays, and finite word length effects.

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