

Enhanced Disturbance Observer-Based Sliding Mode Control for Precise Trajectory Tracking of Liquid-Carrying Robots on Uncertain Terrains

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ABSTRACT- This paper investigates a high-precision trajectory tracking control strategy for differential-drive tanker robots subject to the combined effects of liquid sloshing dynamics and mixed-terrain disturbances. A key challenge in this application arises from the contrasting characteristics of different payload regimes: while a full tank presents maximum total mass and inertia, a half-filled tank (50% partial-load) possesses an expanded free-surface area that facilitates larger wave oscillations (exhibiting peak transient amplitudes near -0.28 m, compared to approximately -0.22 m under full load). To mitigate these state-dependent internal hydrodynamic forces alongside stochastic surface resistance, a control architecture integrating Sliding Mode Control with a Nonlinear Disturbance Observer (DOB-SMC) is formulated. Within this scheme, the observer estimates and compensates for the lumped disturbance composite in real time, thereby allowing a reduction in the discontinuous switching gain to alleviate control chattering. Simulation results along a multi-zone circular trajectory indicate that the DOB-SMC framework consistently attenuates tracking discrepancies across both load conditions, achieving Root Mean Square Error (RMSE) reductions of 42.69% (from 0.0061 m to 0.0035 m) under full load and 33.32% (from 0.0052 m to 0.0035 m) at partial load relative to standalone SMC. Overall, the proposed approach demonstrates practical feasibility for improving trajectory tracking precision and maintaining bounded transient responses during liquid transport over uneven terrains.

Keywords: Tanker Robots, Sliding Mode Control, Disturbance Observer, Liquid Sloshing, Trajectory Tracking.

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1. INTRODUCTION

Over the past decade, Autonomous Mobile Robots (AMRs) have emerged as a cornerstone of industrial automation and smart logistics [1], [2]. Among various logistics applications, the autonomous transport of liquid payloads presents notable challenges [3]. Unlike rigid bodies, liquids exhibit continuously evolving dynamic characteristics. Specifically, acceleration and rapid directional changes induce severe liquid sloshing [4], [5], generating nonlinear inertial forces and disturbing torques that directly act upon the drive system. These destabilizing effects degrade trajectory tracking accuracy and significantly elevate the risk of vehicle rollover in practical operations [6], [7].

Precise motion control of tanker robots is hindered by a dual disturbance mechanism: internal perturbations and external uncertainties. Internally, sloshing dynamics depend heavily on the liquid fill level and tank geometry [8]; as demonstrated by Ibrahim et al. [9], linearized sloshing models often fail when

large-amplitude oscillations occur [10]. Externally, the robot frequently traverses mixed terrains subject to abrupt variations in surface roughness and friction coefficients [11], [12]. Various control paradigms have been explored to address these issues, including adaptive PID [13], Linear Quadratic Regulator (LQR) [14], and Model Predictive Control (MPC) [15]. However, PID control lacks sufficient robustness against substantial disturbances [16], whereas MPC incurs heavy computational burdens that challenge real-time execution on resource-constrained robotic platforms [17]. Sliding Mode Control (SMC) has thus gained traction for its strong disturbance rejection and structural simplicity [18], [19]. Nevertheless, conventional SMC suffers from inherent high-frequency chattering [20], which not only accelerates actuator wear but can also excite the resonant frequencies of the liquid, triggering severe system instability [21].

Although Disturbance Observer (DOB) techniques have been introduced to attenuate chattering in SMC [22], [23], few studies have simultaneously incorporated fill-level-dependent sloshing dynamics while validating performance across complex, mixed-terrain profiles [24], [25]. To address this gap, this paper develops a disturbance-observer-based sliding mode control (DOB-SMC) scheme with the following principal contributions:

Coupled Physical Formulation: A unified dynamic model is established by tightly integrating differential-drive kinematics and actuator electrical dynamics with an equivalent pendulum sloshing model, explicitly capturing the time-varying moment of inertia $J(t)$ induced by the shifting liquid mass.

Dual-Source Active Compensation: An auxiliary-state nonlinear disturbance observer is formulated to reconstruct lumped uncertainties—encompassing internal sloshing reactive torques, external terrain resistance, and parametric inertia mismatches—without requiring angular acceleration measurements. Feedforward compensation enables a reduced SMC switching gain, suppressing chattering while maintaining asymptotic convergence.

Rigorous Validation Across Fill Regimes: The controller's disturbance rejection capability is validated on a circular trajectory spanning three distinct terrain zones (flat, slightly rough, and very rough) under both full-load and 50% partial-load conditions, resolving the contrasting challenges posed by maximum payload mass versus maximum free-surface fluid instability.

The remainder of this paper is organized as follows: Section 2 establishes the system dynamic modeling; Section 3 details the DOB-SMC controller synthesis; Section 4 presents and discusses the simulation results; and Section 5 concludes the paper with directions for future work.

2. SYSTEM MODELING

To design a robust controller, the tanker robot system is decomposed into three component models: Mobile Robot Dynamics, Liquid Sloshing Model, and Electric Drive System Model.

2.1. Mobile Robot Dynamics

Consider a differential-drive mobile robot equipped with two active wheels of radius R , where the track width (distance between the wheels) is denoted by L . The state of the robot is defined by the posture vector $q = [x, y, \theta]^T$. The kinematic equations are described as shown in Equation (1).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (1)$$

where v and ω represent the linear and angular velocities of the robot, respectively. These velocities are derived from the angular velocities of the right wheel (ω_r) and the left wheel (ω_l) as follows as shown in Equation (2).

$$v = \frac{R}{2}(\omega_r + \omega_l); \quad \omega = \frac{R}{L}(\omega_r - \omega_l) \quad (2)$$

2.2. Liquid Sloshing Model

Under planar circular maneuvers, the dominant fluid excitation originates from the resultant horizontal acceleration

$$a_{\text{lumped}} = \sqrt{a_t^2 + a_n^2}, \text{ where } a_t = \dot{v} \text{ and } a_n = v\omega.$$

Following the fundamental mechanical sloshing model developed by Ibrahim [4], a single-degree-of-freedom equivalent pendulum effectively captures the primary free-

surface mode while maintaining real-time computational feasibility.

The fluid oscillation within the tank is characterized by an equivalent pendulum model, where the displacement of the liquid mass x_m is governed by eq. (3), considering the effects of tangential (a_{tan}) and centripetal (a_{cent}) accelerations.

$$\ddot{x}_m + 2\zeta\omega_n\dot{x}_m + \omega_n^2x_m = -a_{acc} \quad (3)$$

where: ω_n denotes the undamped natural frequency of the liquid oscillation; ζ represents the damping ratio of the fluid; a_{acc} is the lumped acceleration acting on the tank, incorporating both tangential and centripetal components.

The dynamic slosh height η , representing the instantaneous fluctuation of the liquid surface, is assumed to be proportional to the displacement x_m as follows eq. (4):

$$\eta = K_s \cdot x_m \quad (4)$$

where K_s is the geometric scaling factor, set to 1.5 in this study based on the tank dimensions.

Crucially, the displacement of the liquid mass x_m induces a time-varying effect on the system's overall moment of inertia. By applying the Parallel Axis theorem, the total payload inertia $J_{payload}$ is formulated as shown eq. (5)

$$J_{payload}(t) = J_{tank} + m_{liquid} \cdot [d + x_m(t)]^2 \quad (5)$$

where J_{tank} is the constant inertia of the empty tank, m_{liquid} is the mass of the fluid, and d is the static distance from the robot's center of rotation to the tank's center. This formulation highlights that $J_{payload}$ is not a constant but a dynamic parameter that fluctuates with the sloshing motion, acting as a significant internal disturbance to the tracking controller.

2.3. Actuator Dynamics

The mobile robot is propelled by DC motors coupled to the driving wheels. The dynamic behavior of the actuation system, which is intrinsically linked to the liquid payload, is described through the angular velocity of the wheels ω_i ($i \in \{r, l\}$) as follows eq. (6):

$$\dot{\omega}_i = -a\omega_i + bU_i - d_i(i \in \{r, l\}) \quad (6)$$

In the above formulation, the parameters a and b are determined by the electromechanical properties of the motors:

$a = \frac{K_t K_b}{J_{actual} R_a}$: represents the electromagnetic damping coefficient; $b = \frac{K_t}{J_{actual} R_a}$ denotes the input gain coefficient;

where: K_t is the motor torque constant ($N.m/A$); K_b is the back-electromotive force (back-EMF) constant ($V.s/rad$); R_a is the armature resistance (Ω) of the DC motor.

The term $J_{actual} = (J_{hat} + J_{payload}) \times 1.1$ represents the effective moment of inertia, where J_{hat} is the nominal inertia of the robot frame and $J_{payload}$ is the time-varying inertia of

the liquid-tank system derived in *section 2.2*. The inclusion of a 10% modeling uncertainty (represented by the factor 1.1) is intentional to evaluate the robustness of the proposed controller. In practical applications, exact identification of the system's inertia is often hindered by measurement errors and mechanical tolerances. By introducing this discrepancy between the nominal model used in the controller design and the actual plant dynamics, we demonstrate the effectiveness of the Disturbance Observer (DOB) in compensating for parameter mismatches.

In *eq. (6)*, U_i is the control input voltage, and d_i represents the lumped disturbance acting on the wheels. This disturbance includes terrain resistance and the reactive inertial forces from the liquid sloshing, formulated as *eq. (7)*:

$$d_i = \frac{\tau_{\text{terrain},i} + \tau_{\text{slosh},i}}{J_{\text{actual}}} \quad (7)$$

where the terrain-induced noise τ_{terrain} is modeled as a stochastic variable, and the sloshing-induced torque τ_{slosh} is approximated by *eq. (8)*:

$$\tau_{\text{slosh}} = m_{\text{liquid}} \cdot a_{\text{slosh}} \cdot h_{\text{arm}} \cdot \sin(2\pi ft) \quad (8)$$

where a_{filt} and f denote the filtered acceleration and the characteristic sloshing frequency, respectively.

3. CONTROLLER DESIGN

The proposed control architecture adopts a cascaded control strategy consisting of two primary loops:

Outer Loop (Kinematic Controller): This stage generates the reference linear and angular velocities required for the robot to follow the desired trajectory.

Inner Loop (Dynamic Controller): This loop employs a Sliding Mode Control (SMC) algorithm integrated with a Disturbance Observer (DOB). The objective of this integrated approach is to ensure that the wheels accurately track the reference velocities, maintaining high precision despite the presence of nonlinear liquid sloshing forces and mixed-terrain disturbances.

The overall schematic of the proposed control system is illustrated in *figure 1*.

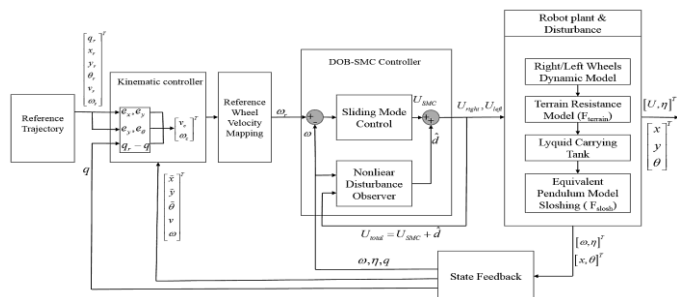


Figure 1. The overall schematic of the proposed control system

3.1. Sliding Mode Controller Synthesis

The Sliding Mode Control (SMC) is synthesized within the inner-loop architecture to ensure that the actual angular velocities of the wheels (ω_r, ω_l) precisely track the reference commands (ω_{rd}, ω_{ld}) generated by the kinematic controller.

SMC is specifically selected for this application due to its inherent robustness and superior capability in rejecting lumped uncertainties. These uncertainties primarily arise from the highly nonlinear dynamics of the tanker robot, the time-varying nature of the liquid payload, and unmodeled actuator disturbances. The fundamental objective of this stage is to drive the velocity tracking errors to the sliding manifold and maintain them there despite the presence of external perturbations.

3.1.1. Sliding Surface Definition

To facilitate the velocity tracking objective for each driving wheel, the tracking error is defined as $e_i = \omega_i - \omega_{id}$. The sliding surface $S_i(t)$ is selected in a first-order form to ensure low computational complexity as Eq. (9)

$$S_i(t) = \omega_i(t) - \omega_{id}(t), \quad i \in \{r, l\} \quad (9)$$

The time-derivative of the sliding surface is then derived as: $\dot{S}_i = \dot{\omega}_i - \dot{\omega}_{id}$

Substituting the actuator dynamics from *eq. (6)* into the above expression yields the following manifold dynamics, as shown *eq. (10)*

$$\dot{S}_i = -a\omega_i + bU_i - d_i - \dot{\omega}_{id} \quad (10)$$

3.1.2. Control Law Design

To derive the control law, the time-derivative of the sliding surface is first obtained by substituting the actuator dynamics from *eq. (6)* into the sliding surface definition, yielding:

$$\dot{S} = (-a\omega + bU - d) - \dot{\omega}_d \quad (11)$$

To ensure that the system state trajectories converge to the sliding manifold $S = 0$, the control input U is synthesized as a combination of equivalent and switching components, as shown in *eq. (12)*:

$$U = U_{eq} + U_{sw} \quad (12)$$

By utilizing the nominal parameters ($\hat{a}, \hat{b}$), the specific control law is formulated in *eq. (13)*:

$$U = \frac{1}{\hat{b}}(\hat{a}\omega + \dot{\omega}_d - \lambda S - K \cdot \text{sat}(S, \Phi)) \quad (13)$$

Substituting the control law from Eq. (13) back into the surface dynamics in Eq. (11), the closed-loop error dynamics

can be analyzed. Here, the saturation function $\text{sat}(S, \Phi)$ is defined as *eq. (14)*:

$$\text{sat}(S, \Phi) = \begin{cases} \frac{S}{\Phi}, & \text{if } |S| \leq \Phi \\ \text{sgn}(S), & \text{if } |S| > \Phi \end{cases} \quad (14)$$

3.1.3. Stability Analysis

To rigorously evaluate the stability of the proposed control law, a positive-definite Lyapunov candidate function $V(t)$ is defined for the wheel angular velocity tracking error as follows:

$$V = \frac{1}{2} S^2 \quad (15)$$

where S is the sliding surface defined in *section 3.2*. To ensure the asymptotic stability and convergence of the system toward the sliding manifold ($S \rightarrow 0$), the time-derivative of the Lyapunov function must satisfy the following condition:

$$\dot{V} = S\dot{S} < 0, \quad \forall S \neq 0 \quad (16)$$

By substituting the actual wheel dynamics $\dot{\omega} = -a\omega + bU - d$ into the sliding surface derivative $\dot{S} = \dot{\omega} - \dot{\omega}_{id}$, we obtain:

$$\dot{V} = S[-a_{act}\omega + b_{act}U - d - \dot{\omega}_{id}] \quad (17)$$

Substituting the proposed control law U from *eq. (12)* into *eq. (17)* yields:

$$\dot{V} = S \left[-a_{act}\omega + \frac{b_{act}}{\hat{b}} (\hat{a}\omega + \dot{\omega}_{id} - \lambda S - K \cdot \text{sat}(S, \Phi)) - d - \dot{\omega}_{id} \right] \quad (18)$$

Assuming that the parameter estimation errors are negligible (*i.e.*, $b_{act} \approx \hat{b}$ and $a_{act} \approx \hat{a}$), the expression can be simplified to:

$$\dot{V} = S[-\lambda S - K \cdot \text{sat}(S, \Phi) - d] = -\lambda S^2 - KS \cdot \text{sat}(S, \Phi) - Sd \quad (19)$$

In the case where the state trajectory is outside the boundary layer ($|S| > \Phi$), the saturation function becomes $\text{sat}(S, \Phi) = \text{sgn}(S)$. Consequently, *eq. (19)* can be rewritten as:

$$\dot{V} = -\lambda S^2 - K|S| - Sd \leq -\lambda S^2 - |S|(K - |d|) \quad (20)$$

To guarantee $\dot{V} < 0$, the switching gain K must be designed to satisfy the condition:

$$K > |d|_{\max} \quad (21)$$

where $|d|_{\max}$ represents the upper bound of the lumped disturbances, including mixed-terrain resistance and sloshing-induced inertial forces. The components of the Lyapunov derivative provide distinct stability features:

The term $-\lambda S^2$ ensures an exponential convergence rate when the tracking error is large.

The term $-(K - |d|)|S|$ ensures robustness, forcing the state trajectory toward the sliding manifold despite the presence of disturbance d .

When the condition in *Eq. (21)* is met, the system achieves Lyapunov stability. The angular velocity error is guaranteed to converge to the boundary layer ($|S| \leq \Phi$) within a finite time.

3.2. Disturbance Observer Design

In physical tanker robot implementations, the lumped wheel disturbance couple's fluid sloshing reaction torques, mixed-terrain friction, and parametric inertia mismatches ($J_{\text{actual}} \neq \hat{J}$) relying exclusively on large SMC switching gains to overcome these disturbances triggers severe chattering, which strains actuators and excites fluid resonance.

To overcome this limitation, a Nonlinear Disturbance Observer (NDO) is synthesized to estimate lumped disturbances online. Active feedforward compensation reduces the required SMC switching gain to cover only residual estimation errors, effectively eliminating chattering while preserving high-precision tracking.

3.2.1. Observer Structure and Formulation

From the actuator dynamics established in *eq. (6)*, the theoretical disturbance entering each driving wheel can be expressed as:

$$d = -a\omega + bU - \dot{\omega} \quad (22)$$

where d represents the total lumped disturbance normalized to angular acceleration units (rad/s^2). Note that d carries a negative sign, capturing the resistive nature of terrain traction loss and the reaction torque induced by liquid sloshing.

However, directly computing d through *eq. (22)* requires evaluating the angular acceleration $\dot{\omega}$. In practical implementations, numerically differentiating raw velocity signals obtained from encoders inevitably amplifies high-frequency measurement noise, which deteriorates tracking performance.

To eliminate the need for angular acceleration measurements, an auxiliary state variable p is introduced. Accounting for the negative disturbance sign in *eq. (6)*, the disturbance estimate $\hat{d}$ is formulated as:

$$\dot{\hat{d}} = p - L_{\text{dob}}\omega \quad (23)$$

where $L_{\text{dob}} > 0$ is the observer gain governing the convergence rate. Differentiating *eq. (23)* with respect to time yields:

$$\dot{\hat{d}} = \dot{p} - L_{\text{dob}}\dot{\omega} \quad (24)$$

Substituting the physical wheel dynamics from *eq. (6)* into *eq. (24)* yields:

$$\dot{\hat{d}} = \dot{p} - L_{\text{dob}}(-a\omega + bU - d) \quad (25)$$

Because the actual disturbance d is inaccessible for direct feedback, the dynamic update law for the auxiliary variable p is synthesized by replacing the unknown d in eq. (25) with its estimated $\hat{d}$:

$$\dot{p} = L_{\text{dob}}(-a\omega + bU - \hat{d}) \quad (26)$$

Substituting Eq. (23) into Eq. (26), the explicit differential equation for the auxiliary variable p is obtained as:

$$\dot{p} = -L_{\text{dob}}p + L_{\text{dob}}(L_{\text{dob}} - a)\omega + L_{\text{dob}}bU \quad (27)$$

3.2.2. Estimation Error and Convergence Analysis

To evaluate the analytical performance of the proposed NDO, the disturbance estimation error is defined as:

$$\tilde{d} = d - \hat{d} \quad (28)$$

Taking the time-derivative of eq. (28) yields:

$$\dot{\tilde{d}} = \dot{d} - \dot{\hat{d}} \quad (29)$$

In practical robotic transport operations, the disturbance dynamics—composed predominantly of low-frequency fluid oscillations and ground roughness variations—evolve slowly dynamics of the observer (*i.e.*, $\dot{\tilde{d}} \approx 0$). Consequently, eq. (29) can be approximated as:

$$\dot{\tilde{d}} = -\dot{\hat{d}} \quad (30)$$

Substituting the time-derivative of the estimated disturbance from eq. (25) into eq. (30) gives:

$$\dot{\tilde{d}} = -[\dot{p} - L_{\text{dob}}(-a\omega + bU - \hat{d})] \quad (31)$$

Substituting the auxiliary variable update law from eq. (26) into eq. (31) results in:

$$\dot{\tilde{d}} = -[L_{\text{dob}}(-a\omega + bU - \hat{d}) - L_{\text{dob}}(-a\omega + bU - d)] \quad (32)$$

Expanding and collecting terms inside the brackets directly cancels out the nominal drive terms ($-a\omega + bU$):

$$\dot{\tilde{d}} = -L_{\text{dob}}[(-\hat{d}) - (-d)] = -L_{\text{dob}}(d - \hat{d}) \quad (33)$$

Substituting the error definition from eq. (28) into eq. (33) yields a linear autonomous first-order differential equation:

$$\dot{\tilde{d}} + L_{\text{dob}}\tilde{d} = 0 \quad (34)$$

The explicit analytical solution to eq. (34) is:

$$\tilde{d}(t) = \tilde{d}(0)e^{-L_{\text{dob}}t} \quad (35)$$

where $\tilde{d}(0) = d(0) - \hat{d}(0)$ denotes the initial estimation discrepancy at $t = 0$ s. Because the observer design gain is

strictly positive ($L_{\text{dob}} > 0$), the pole of the error dynamics lies entirely in the open left-half complex plane. Therefore, Eq. (35) proves that the estimation error $\tilde{d}(t)$ converges to zero at an exponential rate, with the convergence speed directly dictated by L_{dob} .

3.2.3. Integrated Control Law with Disturbance Compensation

To actively reject the lumped dynamic uncertainties without relying on severe switching efforts, the real-time disturbance estimate $\hat{d}$ obtained from eq. (23) is integrated into the sliding mode control framework as an active feedforward compensation term. Recalling the open-loop wheel dynamics from eq. (6) under nominal parameters $\hat{a}$ and $\hat{b}$:

$$\dot{\omega} = -\hat{a}\omega + \hat{b}U - d \quad (36)$$

Using the sliding surface $S = e_{\omega} = \omega_r - \omega$ defined in Eq. (9), the time-derivative of the surface dynamics is expressed as:

$$\begin{aligned} \dot{S} &= \dot{\omega}_r - \dot{\omega} = \dot{\omega}_r - (-\hat{a}\omega + \hat{b}U - d) \\ &= \dot{\omega}_r + \hat{a}\omega - \hat{b}U + d \end{aligned} \quad (37)$$

The composite control input voltage U is formulated by augmenting the nominal sliding mode action U_{SMC} with an active feedforward compensation component U_{comp} :

$$U = U_{\text{SMC}} + U_{\text{comp}} \quad (38)$$

To directly cancel the estimated disturbance $\hat{d}$ within the manifold dynamics eq. (37), the compensation voltage is designed to satisfy $\hat{b}U_{\text{comp}} = \hat{d}$, yielding: relative to the execution bandwidth and fast estimation

$$U_{\text{comp}} = \frac{1}{\hat{b}}\hat{d} \quad (39)$$

Combining eq. (39) with the equivalent and robust switching formulations in eq. (13), the complete integrated control law for each driving wheel is synthesized as

$$U = \frac{1}{\hat{b}}\left[\dot{\omega}_r + \hat{a}\omega + \lambda_s S + K_s \text{sat}\left(\frac{S}{\Phi_s}\right) + \hat{d}\right] \quad (40)$$

Substituting the synthesized control law (40) into the open-loop surface dynamics (37) yields:

$$\begin{aligned} \dot{S} &= \dot{\omega}_r + \hat{a}\omega - \left[\dot{\omega}_r + \hat{a}\omega + \lambda_s S + K_s \text{sat}\left(\frac{S}{\Phi_s}\right) + \hat{d}\right] + d \\ &= -\lambda_s S - K_s \text{sat}\left(\frac{S}{\Phi_s}\right) + (d - \hat{d}) \\ &= -\lambda_s S - K_s \text{sat}\left(\frac{S}{\Phi_s}\right) + \tilde{d} \end{aligned} \quad (41)$$

where $\tilde{d}$ is the residual disturbance estimation error defined in Eq. (28).

To verify closed-loop stability, consider the positive-definite Lyapunov candidate function $V = \frac{1}{2}S^2$. Differentiating V with respect to time outside the boundary layer ($|S| \geq \Phi_s$), where $\text{sat}(S/\Phi_s) = \text{sgn}(S)$ gives:

$$\begin{aligned}\dot{V} = S\dot{S} &= S[-\lambda_s S - K_s \text{sgn}(S) + \tilde{d}] \\ &= -\lambda_s S^2 - K_s |S| + S\tilde{d} \\ &\leq -\lambda_s S^2 - (K_s - |\tilde{d}|)|S|\end{aligned}\quad (42)$$

From eq. (42), to guarantee finite-time reachability and asymptotic convergence ($\dot{V} < 0$), the robust switching gain K_s only needs to satisfy the relaxed condition:

$$K_s > |\tilde{d}|_{\max} \quad (43)$$

This composite structure delivers key advantages over conventional SMC:

Chattering elimination via switching gain reduction: In conventional SMC, the switching gain must overcome the upper bound of the entire lumped disturbance ($K_s > |\tilde{d}|_{\max}$). Because combined sloshing and rugged terrain resistance induce peak disturbances exceeding 50 rad/s², conventional SMC requires an excessively large gain that triggers severe chattering. Under the proposed DOB-SMC, the feedforward term $\hat{d}$ counteracts the bulk of these perturbations, requiring K_s to dominate only the residual error $|\tilde{d}|_{\max}$. Because $\tilde{d}(t) \rightarrow 0$ exponentially according to eq. (35), a modest switching gain ($K_s = 5.0$) fully satisfies eq. (43), eliminating high-frequency control chattering.

Active disturbance rejection: The feedforward term $\frac{1}{b}\hat{d}$ counteracts dynamic sloshing torques and abrupt terrain transitions before they manifest as noticeable tracking deviations, substantially reducing peak transient errors.

Decoupled robust design: The architecture decouples nominal trajectory tracking convergence (dictated by λ_s and boundary layer Φ_s) from the disturbance attenuation capability (governed by the observer bandwidth L_{dob}).

4. SIMULATION RESULTS AND DISCUSSION

4.1. Simulation Environment and Parameter Setup

4.1.1. Simulation Configuration

The proposed control strategy for the tanker robot is implemented and validated using the MATLAB/Simulink 2020b environment. The simulation is conducted with a Fixed-step solver using the ode4 (Runge-Kutta) numerical integration method. The step size is set to $dt = 0.002\text{s}$ (2 ms) to ensure numerical stability and capture the high-frequency dynamics of the liquid sloshing and the sliding mode switching. The total simulation duration is $T_{\text{sim}} = 40\text{s}$.

4.1.2. System and Control Parameters

The system performance is evaluated based on the practical physical parameters of the tanker robot and optimized control gains. These parameters are categorized into three main groups: mechanical system properties (table 1), motor specifications (table 2), and control parameters (tables 3).

Table 1. Physical Parameters of the Tanker Robot and Liquid System

Symbol	Parameter	Value	Unit
R	Wheel radius	0.05	m
L	Distance between wheels	0.25	m
m_{tank}	Mass of the empty tank	1.0	kg
m_{liquid}	Liquid mass (Full)	2.0	kg
R_{tank}	Tank radius	0.1	m
G	Gravitational acceleration	9.81	m/s ²
ω_n	Natural frequency of sloshing	3.14	rad/s
ζ	Liquid damping ratio	0.05	-

Table 2. Actuator Specifications and Model Uncertainties

Symbol	Parameter	Actual Value	Nominal Value
R_a	Armature resistance	1.0	1.0
K_t	Torque constant	0.5	0.5
K_b	Back-EMF constant	0.5	0.5
J_{hat}	Basic moment of inertia	-	0.01
J_{actual}	Actual moment of inertia ($J_{\text{hat}} + J_{\text{payload}}$) $\times 1.1$	-	-

Table 3. Controller Gains and Parameter

Controller	Symbol	Description	Value
<i>Kinematic</i>	k_x, k_y, k_{θ}	Position and orientation gains	2.0, 5.0, 4.0
<i>SMC</i>	λ_s, K_s, Φ_s	Sliding gain, Robust gain, Boundary layer	5.0, 5.0, 0.8
<i>DOB</i>	L_{dob}	Disturbance observer gain	180

In addition to actuator and structural dynamics, the terrain-induced disturbance is mathematically modeled as a space-dependent piecewise function, $F_{\text{terrain}}(x, y)$, which varies according to the robot's instantaneous spatial coordinates within the operational environment:

$$F_{\text{terrain}}(x, y) = \begin{cases} d_{\text{flat}}(t), & \text{for } (x, y) \in \text{Flat Zone} \\ d_{\text{slight}}(t), & \text{for } (x, y) \in \text{Slightly Rough Zone} \\ d_{\text{rough}}(t), & \text{for } (x, y) \in \text{Very Rough Zone} \end{cases}$$

where $d_{\text{flat}}(t)$, $d_{\text{slight}}(t)$, and $d_{\text{rough}}(t)$ denote stochastic resistance torques characterized by zero-mean Gaussian noise

with distinct variances and bounded amplitudes corresponding to surface roughness. The coexistence of high-amplitude terrain noise and nonlinear liquid sloshing forces creates a rigorous benchmark environment. This combination poses a significant challenge for trajectory tracking, necessitating a control strategy with high-degree robustness to maintain stability under such extreme operational conditions.

To rigorously evaluate the robustness and adaptability of the proposed DOB-SMC architecture, the simulation is conducted under two distinct and challenging payload conditions. The first scenario considers the robot operating under a full-load condition, which serves as a critical benchmark to ensure the controller can suppress high-amplitude oscillations induced by the maximum payload when navigating through mixed terrains. The second scenario explores the partial-load condition (50% capacity), a state that is theoretically more taxing than the full-tank case. Due to the larger free-surface area available in a half-full tank, the system is subjected to highly nonlinear and aggressive sloshing effects. By testing the robot across these two extremes—maximum inertia in the full-tank case and maximum fluid instability in the partial-fill case—this study aims to verify the controller's ability to maintain precise trajectory tracking and effectively mitigate time-varying moments of inertia under volatile operational environments.

4.2. Performance Analysis

4.2.1. Trajectory Tracking Accuracy

To visually assess the tracking capabilities, *figure 2* illustrates the x -plane circular trajectory results under mixed-terrain and distinct payload conditions. Despite the high-mass payload in the full-load condition and aggressive sloshing in the partial-load case, the proposed DOB-SMC architecture ensures negligible deviations from the reference path. In striking contrast, the standard SMC exhibits pronounced tracking errors, particularly when navigating the "Slightly rough" and "Very rough" terrain zones, confirming that the observer-based compensation is essential for neutralizing robust nonlinear disturbances.

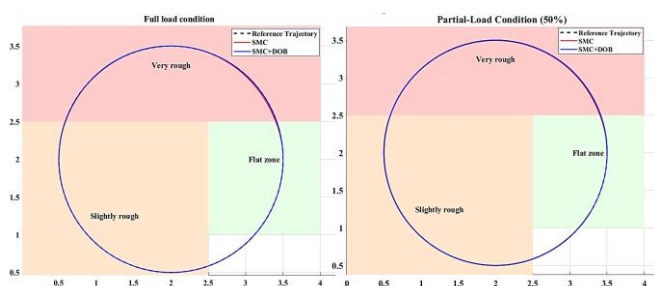


Figure 2. Comparative trajectory tracking performance in the XY-plane for a circular path under mixed-terrain conditions. Comparison between standard SMC (red) and the proposed DOB-SMC (blue) under full-load (left) and partial-load (50%) (right) conditions

The time-varying robot tracking error profiles for both payload scenarios are presented in *figure 3*, providing a clear comparison of the transient and steady-state performance between the two control strategies.

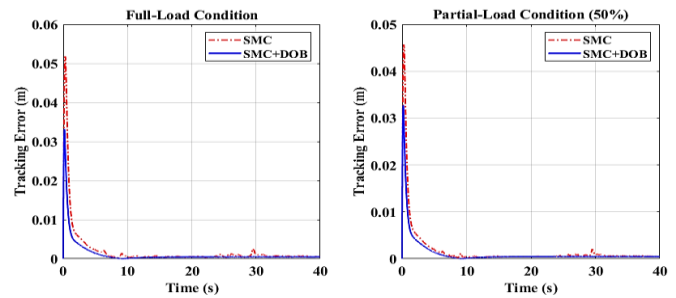


Figure 3. Performance comparison of trajectory tracking errors for a tanker robot. Left: Full load. Right: Partial load (50%), demonstrating error reduction by DOB compensation

During the initial transient phase ($t < 10$ s), both control strategies effectively drive the tracking error toward zero; however, the SMC+DOB configuration achieves a significantly lower initial error peak (peaking around 0.033 m under full load compared to roughly 0.052 m for baseline SMC) and converges noticeably faster. Furthermore, beyond 10 seconds, the integrated SMC+DOB controller maintains a smoother, near-zero steady-state profile, effectively suppressing the minor residual fluctuations observed in the standalone SMC response around 30 seconds. While the baseline SMC controller demonstrates acceptable convergence, these quantitative enhancements clearly highlight the role of the disturbance observer in reducing transient overshoot and improving trajectory tracking accuracy across different payload conditions.

4.2.2. Disturbance Estimation Performance

The performance of the Nonlinear Disturbance Observer (NDO) is critically analyzed to verify its capability in estimating the complex lumped disturbances. Figure 4 illustrates the comparison between the actual disturbances (including terrain resistance and sloshing forces) and the estimated values provided by the DOB. It can be observed that the observer achieves high-fidelity tracking of the disturbance profiles in both scenarios. Even during sharp transitions into the 'Very Rough' zones, the estimation error $\hat{d}$ remains within a narrow bound after a very brief transient period. This high-performance estimation is the key factor that allows the SMC to maintain superior trajectory precision with reduced chattering.

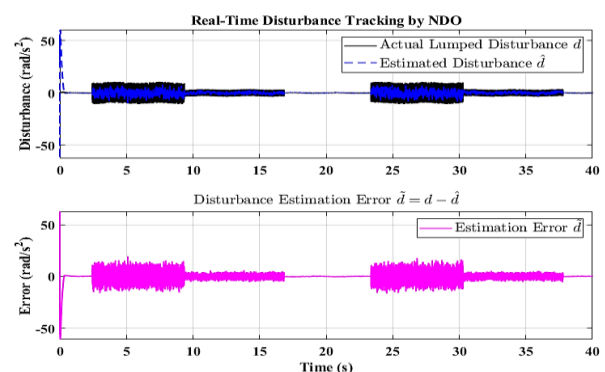


Figure 4. Disturbance estimation performance of the proposed NDO. Top: Real-time tracking of the lumped disturbances (d vs $\hat{d}$). Bottom: Corresponding estimation error ($\hat{d}$) under full-load (left) and partial-load (50%) (right) conditions

Figure 4 presents the real-time disturbance tracking performance of the proposed NDO, displaying both the actual versus estimated lumped disturbances on the top subplot and the corresponding estimation error ($\tilde{d}$) on the bottom subplot. Throughout the operational timeline, the estimated disturbance closely tracks the fluctuations of the actual lumped disturbance, maintaining a high level of fidelity across varying load conditions. The estimation error remains tightly bounded near zero for most of the duration, with noticeable high-frequency oscillations appearing only during active disturbance intervals (approximately between 2.5–9.5 seconds and 23–30.5 seconds). Although these transient peaks reach up to roughly $\pm 15 \text{ rad/s}^2$ during active phases, the error quickly dampens and stabilizes, reflecting the robust tracking capability and overall reliability of the proposed observer structure.

4.2.3. Liquid Sloshing Suppression

To analyze the effect of the liquid payload on vehicle stability, the sloshing dynamics are recorded. Figure 5 displays the deviations in liquid height and wave amplitude within the tank, illustrating the transient fluid behavior during the robot's movement.

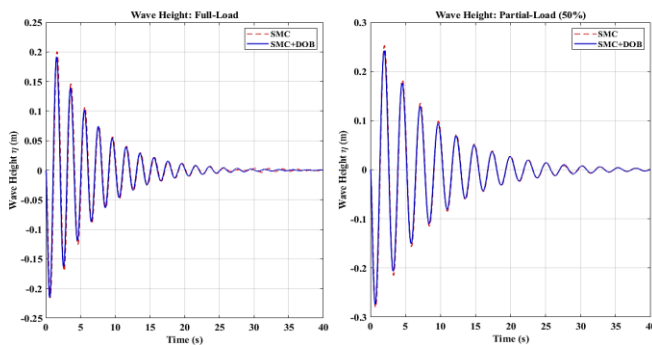


Figure 5. Fluid dynamic response within the container

Figure 5 illustrates the fluid dynamic response in terms of wave height (η) over time, comparing the performance of the conventional SMC and the integrated SMC+DOB controllers under full-load and partial-load (50%) conditions. Across both scenarios, sloshing amplitudes exhibit a damped oscillatory behavior, with the partial-load condition demonstrating a slightly higher peak displacement (reaching nearly -0.28 m compared to roughly -0.22 m under full load). Notably, incorporating the disturbance observer (SMC+DOB) achieves a modest yet visible reduction in peak wave amplitudes during the initial high-transient cycles (0 to 5 seconds) compared to standalone SMC. While the overall trajectories remain closely matched due to the dominant underlying sloshing dynamics, these subtle gains indicate that the DOB actively contributes to attenuating disturbance effects and promoting surface stabilization.

To provide a spatial perspective on the fluid-structure interaction, the three-dimensional evolution of the liquid surface is analyzed. Figure 6 illustrates the 3D sloshing height profiles along the robot's trajectory, highlighting how the

proposed DOB-SMC maintains stability despite the continuous shifts in the liquid's center of mass.

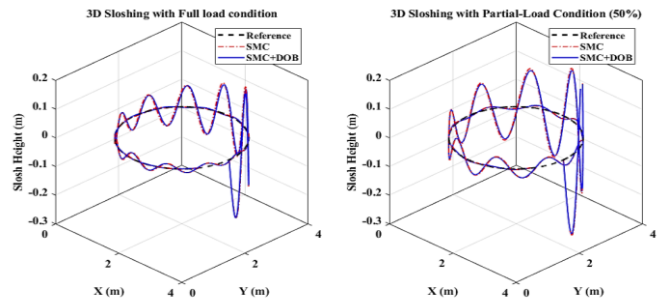


Figure 6. 3D visualization of sloshing height variations during trajectory tracking. Left: Full-load condition. Right: Partial-load (50%) condition

4.2.4. Sliding Manifold Analysis

To verify the reachability and stability of the proposed controller, the behavior of the sliding surfaces (S) for both wheels is analyzed. Figure 7 illustrates the sliding surface trajectories under the two simulated scenarios.

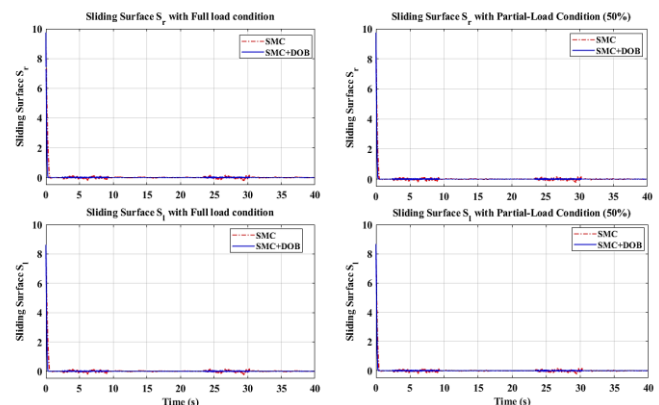


Figure 7. Time-domain response of the sliding surfaces for left and right wheels. Comparison between standard SMC and the proposed DOB-SMC under Full-Load (left) and Partial-Load (right) conditions

As observed in figure 7, in all examined cases, both control architectures drive the sliding surfaces toward zero within approximately one second, demonstrating rapid reaching dynamics and system stability regardless of payload variations. Throughout the subsequent sliding phase, the integration of the disturbance observer ensures that S_r and S_l stay tightly regulated at zero, effectively suppressing the visible ripples observed in the baseline SMC around the intervals of 5--10 s and 25--30 s. While the standalone SMC maintains adequate boundedness across the entire timeline, the smoother trajectory and reduced chattering achieved by DOB-SMC reasonably confirm the observer's capability to mitigate matched disturbances without compromising convergence speed.

4.2.5. Statistical Error and Improvement

To quantify the tracking performance enhancement, the Root Mean Square Error (RMSE) of the Cartesian trajectory is calculated. Figure 8 presents a comparative analysis of the RMSE values, demonstrating a substantial reduction in

tracking errors when utilizing the proposed DOB-based compensation.

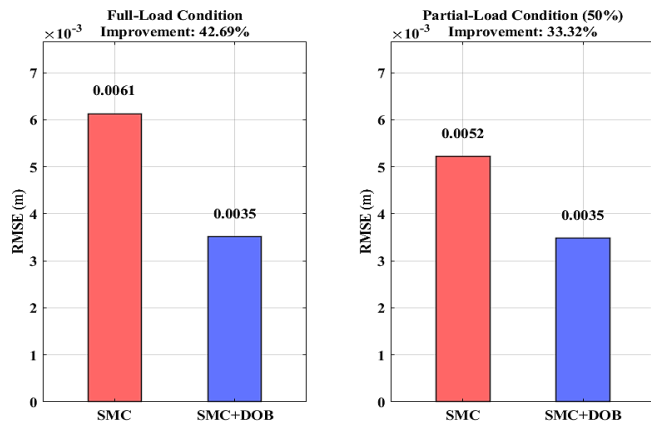


Figure 8. Quantitative performance comparison using Root Mean Square Error (RMSE) of the Cartesian trajectory

Figure 8 provides a quantitative assessment of trajectory tracking performance using the Root Mean Square Error (RMSE) across the Cartesian path, comparing standalone SMC against the proposed SMC+DOB under full-load and 50% partial-load conditions. Under full-load operation, the integration of the disturbance observer reduces the tracking error from 0.0061 m to 0.0035 m, yielding a substantial accuracy improvement of 42.69%. Similarly, for the partial-load case, the RMSE declines from 0.0052 m with standard SMC down to 0.0035 m with SMC+DOB, reflecting a solid 33.32% error reduction. Although standard SMC maintains acceptable path-following precision within sub-centimeter bounds, these consistent reductions under both operating regimes demonstrate the observer's capability to mitigate lumped uncertainties and enhance overall motion repeatability.

4.3. Discussion

The comparative simulation results illustrate the feasibility and performance characteristics of integrating a disturbance observer into the sliding mode control framework for tanker robots. **Tracking Precision and Disturbance Compensation:** The quantitative evaluation in figure 8 indicates that the DOB-SMC approach leads to noticeable reductions in tracking errors under both full-load and partial-load conditions, yielding improvements of 42.69% and 33.32%, respectively. By estimating the lumped disturbances—encompassing stochastic ground roughness, parametric mismatches (including a 10% inertia discrepancy), and reaction torques—the observer compensates for major perturbation components, thereby mitigating their impact on path tracking accuracy. This feedforward compensation reduces reliance on high switching gains, which substantially alleviates high-frequency chattering along the sliding manifold ($S_r, S_l \rightarrow 0$).

Fluid Dynamics and Physical Constraints: As reflected in figures 5 and fig. 6, the reduction in sloshing wave amplitude achieved by the integrated scheme remains modest, though observable during the initial transient phase (0–5 s). It is important to acknowledge that while the DOB-SMC mitigates

abrupt chassis accelerations that contribute to fluid excitation, the primary sloshing modes are fundamentally governed by tank geometry, fill ratio, and natural fluid resonance frequencies. Consequently, wheel-level motion control alone offers bounded surface attenuation; further suppression of sloshing dynamics would likely require complementary measures, such as internal baffles or specialized liquid-damping mechanisms.

Computational and Practical Considerations: The observer formulation utilizes wheel angular velocity feedback without requiring direct acceleration measurements, thereby avoiding the numerical differentiation noise typically encountered in encoder-based setups. Although high-frequency residual estimation errors temporarily increase during abrupt terrain transitions (reaching transient peaks near $\pm 15 \text{ rad/s}^2$ in Figure 4), the closed-loop system quickly restores convergence. These observations suggest that the proposed architecture provides a practical and balanced trade-off between disturbance attenuation and robust tracking under uneven operating conditions.

5. CONCLUSIONS

This paper has presented and evaluated a Disturbance Observer-Based Sliding Mode Control (DOB-SMC) scheme for differential-drive tanker robots navigating uneven terrains. By coupling vehicle dynamics with an equivalent pendulum sloshing model and stochastic ground resistance, the proposed framework effectively addresses internal hydrodynamic moments alongside external disturbances.

The primary findings of this study include:

Payload Adaptability: Despite larger wave oscillations in the 50% partial-load scenario compared to the full-load case, the controller maintains consistent tracking stability within sub-centimeter bounds across both fill conditions

Improved Tracking Precision: Active disturbance compensation reduces trajectory RMSE by 42.69% (from 0.0061 m to 0.0035 m) under full load and 33.32% (from 0.0052 m to 0.0035 m) under partial load relative to baseline SMC.

Chattering Alleviation: Feedforward compensation permits a smaller switching gain, noticeably mitigating high-frequency control chattering on the sliding surfaces while maintaining robustness against a 10% inertia mismatch.

Future work will focus on experimental validation on a physical robotic platform to evaluate embedded execution constraints and real-world fluid–structure interactions.

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and Do Khac Tiep; writing—review and editing, Bui Van Tu and Do Khac Tiep. All authors have read and agreed to the published version of the manuscript.

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